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Appendix to:
A Model for International Spillovers
to Emerging Markets

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Appendix A: model's stationary FOC

Here we present the First Order Conditions (FOC) from savers, entrepreneurs and the firms. Rule of thumb households do not optimise but set their wage and effort to the economy-wide average and consume their total income. FOC from the financial sector are presented in the core of the paper (we refer to [Bernanke et al. \(1999\)](#) for the details on the agency problem). The Central Bank and Government follow simple rules and do not optimise. Those FOCs have been expressed following the convention that lower case variables denote the stationarized equivalent of their upper case counterparts. In particular, some variables have a nominal and real trend because of the positive average labour productivity growth rate and the positive central bank's inflation target. These variables are stationarized following [Adolfson et al. \(2007\)](#) such that $x_t = \frac{X_t}{z_t}$ for real variables and $x'_t = \frac{X'_t}{P_t z_t}$ for nominal variables.¹

A.1 Households

The consumption demand functions for the domestic and the imported goods are given by maximising the consumption basket under a fixed consumption spendings:

$$c_t^d = (1 - \varepsilon_{m,t}\omega_c) \left[\frac{P_t}{P_t^C} \right]^{-\eta_c} c_t, \quad (1)$$

$$c_t^m = \varepsilon_{m,t}\omega_c \left[\frac{P_t^m}{P_t^c} \right]^{-\eta_c} c_t, \quad (2)$$

where P_t is the domestic good price, P_t^m the imported good price and P_t^c represents the Consumer Price Index (CPI) and is given by:

$$P_t^c = [(1 - \varepsilon_{m,t}\omega_c)(P_t)^{1-\eta_c} + \varepsilon_{m,t}\omega_c(P_t^m)^{1-\eta_c}]^{1/(1-\eta_c)}. \quad (3)$$

A.1.1 Savers

Savers work, consume and buy bonds. They maximise their utility subject to exogenous habits in consumption with respect to domestic and foreign bonds holding and consumption. The first order conditions associated to savers with shadow value v_t are given by

$$w.r.t. C_t^s : \frac{1}{(c_t^s - bc_{t-1}^s/\mu_z)^{\sigma_c}} - \psi_{z,t} \frac{P_t^c}{P_t} (1 + \tau^c) = 0 \quad (4)$$

$$w.r.t. B_{t+1} : -\psi_{z,t} + \beta_S E_t \frac{\psi_{z,t+1}}{\mu_z \pi_{t+1}} (\varepsilon_{b,t} R_t - \tau^k (\varepsilon_{b,t} R_t - 1)) = 0 \quad (5)$$

$$w.r.t. B_{t+1}^* : -\psi_{z,t} S_t + \beta_S E_t \frac{\psi_{z,t+1}}{\mu_z \pi_{t+1}} \left(S_{t+1} \varepsilon_{b,t} R_t^* \Phi(a_t, \tilde{\phi}_t^a) - \tau^k S_{t+1} (\varepsilon_{b,t} R_t^* \Phi(a_t, \tilde{\phi}_t^a) - 1) - \tau^k (S_{t+1} - S_t) \right) = 0. \quad (6)$$

¹ [Adolfson et al. \(2007\)](#) draw on [Altig et al., 2003](#).

where $\pi_t = \frac{P_t}{P_{t-1}}$, $\mu_z = \frac{z_t}{z_{t-1}}$, $c_t^s = \frac{C_t^s}{z_t}$, $\psi_{z,t} = v_t z_t P_t$ and $a_t = \frac{A_t}{z_t}$.

Country risk premium Combining the FOC with respect to domestic and foreign bonds gives the uncovered interest rate parity (UIP) condition

$$R_t = R_t^* \Phi\left(\frac{A_t}{z_t}, \tilde{\phi}_t^a\right) E_t \frac{S_{t+1}}{S_t} \quad (7)$$

This equality shows that the spread between domestic and foreign nominal risk free rates depends on the anticipated domestic currency depreciation, the country-wide foreign debt and an UIP shock.

Wage setting Each household has a probability $(1 - \xi_w)$ to be allowed to optimally reset the nominal wage. Otherwise, the wage is indexed on previous period consumer price inflation π_{t-1}^c , the Central Bank inflation target $\bar{\pi}$ and to hourly labour productivity growth. Households that can re-optimize their wage maximize

$$\sum_{s=0}^{\infty} (\beta \xi_w)^s \left(v_{t+s} \frac{1 - \tau^y}{1 + \tau^w} W_{j,t+s} h_{j,t+s} - A_h \frac{(h_{j,t+s})^{1+\sigma_h}}{1 + \sigma_h} \right)$$

where

$$\begin{aligned} W_{j,t+s} &= W_{j,t}^{new} (\pi_t^c \dots \pi_{t+s-1}^c)^{\kappa_w} \left(\frac{\Delta y_t}{\Delta H_t} \dots \frac{\Delta y_{t+s-1}}{\Delta H_{t+s-1}} \right)^{\kappa_w} \bar{\pi}^{(1-\kappa_w)s} \\ h_{j,t+s} &= \left(\frac{W_{j,t+s}}{W_{t+s}} \right)^{-\epsilon_w} H_{t+s}^S \\ &= \left(\frac{W_{j,t}^{new} (\pi_t^c \dots \pi_{t+s-1}^c)^{\kappa_w} \left(\frac{\Delta y_t}{\Delta H_t} \dots \frac{\Delta y_{t+s-1}}{\Delta H_{t+s-1}} \right)^{\kappa_w} \bar{\pi}^{(1-\kappa_w)s}}{W_{t+s}} \right)^{-\epsilon_w} H_{t+s}^S \end{aligned}$$

with respect to the new wage W_t^{new} . Note that v_{t+s} is the Lagrange multiplier in the household optimisation problem and that it is also useful to define

$$\begin{aligned} \Pi_{t,t+s-1}^c &= (\pi_t^c \dots \pi_{t+s-1}^c) \\ \Delta_{t,t+s-1}^{y/h} &= \left(\frac{\Delta y_t}{\Delta H_t} \dots \frac{\Delta y_{t+s-1}}{\Delta H_{t+s-1}} \right) \\ \Pi_{t+1,t+s} &= (\pi_{t+1} \dots \pi_{t+s}) \end{aligned}$$

Rearranging using the above equations gives:

$$\begin{aligned} & \sum_{s=0}^{\infty} (\beta \xi_w)^s \left[v_{t+s} \frac{1 - \tau^y}{1 + \tau^w} W_{j,t}^{new} \left(\Pi_{t,t+s-1}^c \Delta_{t,t+s-1}^{y/h} \right)^{\kappa_w} \bar{\pi}^{(1-\kappa_w)s} \right. \\ & \times \left(\frac{W_{j,t}^{new} \left(\Pi_{t,t+s-1}^c \Delta_{t,t+s-1}^{y/h} \right)^{\kappa_w} \bar{\pi}^{(1-\kappa_w)s}}{W_{t+s}} \right)^{-\epsilon_w} H_{t+s}^S \\ & \left. - \frac{A_h}{1 + \sigma_h} \left(\frac{W_{j,t}^{new} \left(\Pi_{t,t+s-1}^c \Delta_{t,t+s-1}^{y/h} \right)^{\kappa_w} \bar{\pi}^{(1-\kappa_w)s}}{W_{t+s}} \right)^{-(1+\sigma_h)\epsilon_w} (H_{t+s}^S)^{1+\sigma_h} \right] \end{aligned}$$

Expressing it in term of real wage and simplifying gives:

$$\begin{aligned} & \sum_{s=0}^{\infty} (\beta \xi_w)^s \left[\psi_{t+s} \frac{1 - \tau^y}{1 + \tau^w} \left(\frac{\bar{w}_{j,t}^{new} \left(\Pi_{t,t+s-1}^c \Delta_{t,t+s-1}^{y/h} \right)^{\kappa_w} \bar{\pi}^{(1-\kappa_w)s}}{\Pi_{t+1,t+s}} \right)^{1-\epsilon_w} (\bar{w}_{t+s})^{\epsilon_w} H_{t+s}^S \right. \\ & \left. - \frac{A_h}{1 + \sigma_h} \left(\frac{\bar{w}_{j,t}^{new} \left(\Pi_{t,t+s-1}^c \Delta_{t,t+s-1}^{y/h} \right)^{\kappa_w} \bar{\pi}^{(1-\kappa_w)s}}{\bar{w}_{t+s} \Pi_{t+1,t+s}} \right)^{-(1+\sigma_h)\epsilon_w} (H_{t+s}^S)^{1+\sigma_h} \right] \end{aligned}$$

The FOC is now easy to derive and reads:

$$\begin{aligned} & (\epsilon_w - 1) (\bar{w}_{j,t}^{new})^{-\epsilon_w} \sum_{s=0}^{\infty} (\beta \xi_w)^s \psi_{t+s} \frac{1 - \tau^y}{1 + \tau^w} \left(\frac{\left(\Pi_{t,t+s-1}^c \Delta_{t,t+s-1}^{y/h} \right)^{\kappa_w} \bar{\pi}^{(1-\kappa_w)s}}{\Pi_{t+1,t+s}} \right)^{1-\epsilon_w} (\bar{w}_{t+s})^{\epsilon_w} H_{t+s}^S \\ & = \epsilon_w (\bar{w}_{j,t}^{new})^{-(1+\sigma_h)\epsilon_w - 1} \sum_{s=0}^{\infty} (\beta \xi_w)^s A_h \left(\frac{\left(\Pi_{t,t+s-1}^c \Delta_{t,t+s-1}^{y/h} \right)^{\kappa_w} \bar{\pi}^{(1-\kappa_w)s}}{\bar{w}_{t+s} \Pi_{t+1,t+s}} \right)^{-(1+\sigma_h)\epsilon_w} (H_{t+s}^S)^{1+\sigma_h} \end{aligned}$$

which simplifies to:

$$(\bar{w}_t^{new})^{1+\sigma_h\epsilon_w} = \frac{\epsilon_w}{\epsilon_w - 1} \frac{\sum_{s=0}^{\infty} (\beta \xi_w)^s A_h \left(\frac{\left(\Pi_{t,t+s-1}^c \Delta_{t,t+s-1}^{y/h} \right)^{\kappa_w} \bar{\pi}^{(1-\kappa_w)s}}{\bar{w}_{t+s} \Pi_{t+1,t+s}} \right)^{-(1+\sigma_h)\epsilon_w} (H_{t+s}^S)^{1+\sigma_h}}{\sum_{s=0}^{\infty} (\beta \xi_w)^s \psi_{t+s} \frac{1 - \tau^y}{1 + \tau^w} \left(\frac{\left(\Pi_{t,t+s-1}^c \Delta_{t,t+s-1}^{y/h} \right)^{\kappa_w} \bar{\pi}^{(1-\kappa_w)s}}{\Pi_{t+1,t+s}} \right)^{1-\epsilon_w} (\bar{w}_{t+s})^{\epsilon_w} H_{t+s}^S}$$

since all re-optimising households set the same wage. This last equation is the wage-Phillips curve with partial indexation. In Dynare, the infinite sum can be rewritten as a set of three equations:

$$(\bar{w}_t^{new})^{1+\sigma_h\epsilon_w} = \left(\frac{\epsilon_w}{\epsilon_w - 1} \right) \frac{X_{1,t}^H}{X_{2,t}^H} \quad (8)$$

$$X_{1,t}^H = A_h \bar{w}_t^{(1+\sigma_h)\epsilon_w} (H_t^S)^{1+\sigma_h} + \beta \xi_w \left(\frac{\left(\Pi_t^c \frac{\Delta y_t}{\Delta H_t} \right)^{\kappa_w} \bar{\pi}^{(1-\kappa_w)}}{\pi_{t+1}} \right)^{-(1+\sigma_h)\epsilon_w} E_t X_{1,t+1}^H \quad (9)$$

$$X_{2,t}^H = \frac{1 - \tau^y}{1 + \tau^w} \psi_t \bar{w}_t^{\epsilon_w} H_t^S + \beta \xi_w \left(\frac{\left(\pi_t^c \frac{\Delta y_t}{\Delta H_t} \right)^{\kappa_w} \bar{\pi}^{(1-\kappa_w)}}{\pi_{t+1}} \right)^{-(\epsilon_w-1)} E_t X_{2,t+1}^H \quad (10)$$

Labour packer The real wage index evolves according to

$$\bar{w}_t^{1-\epsilon_w} = (1 - \xi_w) (\bar{w}_t^{new})^{1-\epsilon_w} + \xi_w \left(\frac{\left(\pi_{t-1}^c \frac{\Delta y_{t-1}}{\Delta H_{t-1}} \right)^{\kappa_w} \bar{\pi}^{1-\kappa_w} \bar{w}_{t-1}}{\pi_t} \right)^{1-\epsilon_w} \quad (11)$$

Labour mobility The labour allocation problem gives primary and secondary sectors demands for labour

$$\bar{w}_t^f = \left[\frac{H_t^f}{(1 - \omega_h) H_t} \right]^{1/\eta_h} \bar{w}_t, \quad (12)$$

$$\bar{w}_t^p = \left[\frac{H_t^p}{\omega_h H_t} \right]^{1/\eta_h} \bar{w}_t, \quad (13)$$

that link the sectoral wage to sectoral demand and $\bar{w}_t = \frac{W_t}{P_t z_t}$; $\bar{w}_t^p = \frac{W_t^p}{P_t z_t}$ and $\bar{w}_t^f = \frac{W_t^f}{P_t z_t}$

A.1.2 Entrepreneurs

Entrepreneurs consume, invest in capital used in the mining and manufactured sectors, manage the firms and borrow from the bank. For simplicity as well as in order to ease the comparison with [Adolfson et al. \(2007\)](#), we decided to split the impatient household problem. The first problem presented here focuses on consumption, investment and borrowing decisions while the firms production (choosing inputs composition in order to minimise production costs) and price setting will be presented in the firms section.

Each entrepreneur j maximises her utility with respect to consumption and borrowing with shadow value v_t^e on the budget constraint:

$$\begin{aligned} w.r.t. C_t^e & : \frac{1}{(c_t^e - b c_{t-1}^e / \mu_z)^{\sigma_c}} - \psi_{z,t}^e \frac{P_t^c}{P_t} (1 + \tau^c) = 0 \\ w.r.t. B_{t+1}^e & : -\psi_{z,t}^e + \beta_E E_t \frac{\psi_{z,t+1}^e}{\mu_z \pi_{t+1}} (R_t^L - \tau^k (R_t^L - 1)) = 0 \end{aligned} \quad (14)$$

where we have used the same stationary technique as for the patient households.

Entrepreneurs also maximise their utility with respect to the capital stock, investment

and capital utilisation rate in sector q with shadow value $\omega_t^{e,q}$ on capital q accumulation rule

$$w.r.t. \Delta_t : \psi_{z,t}^e \frac{P_t^{k,q}}{z_t P_t} = \omega_t^{e,q} \quad (15)$$

$$w.r.t. k_{t+1}^q : \psi_{z,t}^e \frac{P_t^{k,q}}{P_t} = \beta_E \frac{\psi_{z,t+1}^e}{\mu_z} \left((1 - \tau^k) r_{t+1}^{k,q} + (1 - \delta) \frac{P_{t+1}^{k,q}}{P_{t+1}} - a(u_{t+1}^q) \right) \quad (16)$$

$$w.r.t. i_t^q : -\psi_{z,t}^e \frac{P_t^i}{P_t} + \frac{P_t^{k,q}}{P_t} \psi_{z,t}^e \Upsilon_t \left(1 - \tilde{S} \left(\frac{\mu_z i_t^q}{i_{t-1}^q} \right) - \tilde{S}' \left(\frac{\mu_z i_t^q}{i_{t-1}^q} \right) \frac{\mu_z i_t^q}{i_{t-1}^q} \right) \\ + \beta_E E_t \left(\frac{P_{t+1}^{k,q}}{P_{t+1}} \frac{\psi_{z,t+1}^e}{\mu_z} \Upsilon_{t+1} \tilde{S}' \left(\frac{\mu_{z,t+1} i_{t+1}^q}{i_t^q} \right) \left(\frac{\mu_z i_{t+1}^q}{i_t^q} \right)^2 \right) = 0 \quad (17)$$

$$w.r.t. u_t^q : \psi_{z,t}^e \left((1 - \tau^k) r_t^{k,q} - a'(u_t^q) \right) = 0 \quad (18)$$

where $r_t^k \equiv \frac{R_t^k}{P_t}$ is the rental rate of capital corresponding to marginal productivity of capital and $i_t = \frac{I_t}{z_t}$.

Entrepreneurs (aggregated) budget constraint is important since it determines their borrowing need (whose aggregate value will influence the lending rate). Its stationary form reads

$$\frac{P_t^c}{P_t} c_t^e (1 + \tau^c) + \frac{P_t^i}{P_t} (i_t^p + i_t^f) + R_{t-1}^L \frac{b_t^e}{\pi_t \mu_z} \\ = (1 - \tau^k) \Pi_t + \tau^k (R_{t-1}^L - 1) \frac{b_t^e}{\pi_t \mu_z} + b_{t+1}^e + t r_t^e, \quad (19)$$

where

$$\Pi_t = y_t^f + \frac{(S_t P_t^x - (1 - \omega_x) P_t - \omega_x P_t^m)}{P_t} x_t^f + \frac{(S_t P_t^{*p} - \omega_x P_t^m)}{P_t} x_t^p \\ - R_{t-1}^L \left(\bar{w}_t^p H_t^p + \bar{w}_t^f H_t^f + \frac{P_t^m}{P_t} n_t^m \right) - \frac{\phi}{P_t} \quad (20)$$

Capital Accumulation We can also rewrite the capital accumulation rule in stationary form as

$$\bar{k}_{t+1} = (1 - \delta) \frac{\bar{k}_t}{\mu_z} + \Upsilon_t \left(1 - \tilde{S} \left(\frac{\mu_z i_t}{i_{t-1}} \right) \right) i_t \quad (21)$$

by using

$$\bar{k}_{t+1} \equiv \frac{\bar{K}_{t+1}}{z_t} \quad (22)$$

Investment Basket The two investment demand functions are given by maximising the investment basket under a fixed investment spending:

$$i_t^{d,q} = (1 - \varepsilon_{m,t} \omega_i) \left[\frac{P_t}{P_t^i} \right]^{-\eta_i} i_t^q, \quad (23)$$

$$i_t^{m,q} = \varepsilon_{m,t} \omega_i \left[\frac{P_t^m}{P_t^i} \right]^{-\eta_i} i_t^q, \quad (24)$$

where P_t^m is the price of the imported good and P_t^i is the aggregate investment price² given by:

$$P_t^i = \left[(1 - \varepsilon_{m,t} \omega_i) (P_t)^{1-\eta_i} + \varepsilon_{m,t} \omega_i (P_t^m)^{1-\eta_i} \right]^{1/(1-\eta_i)} \quad (25)$$

A.1.3 Rule of thumb households

The stationary rule of thumb households budget constraint reads

$$(1 + \tau_t^c) \frac{P_t^c}{P_t} c_t^r = \frac{1 - \tau_t^y}{1 + \tau_t^w} \bar{w}_t H_t^r + t r^r \quad (26)$$

where $\bar{w}_t = \frac{W_t}{P_t z_t}$

A.2 Firms

Here we present the profit maximisation problem of the firms in the commodity and manufacturing sectors.

A.2.1 Commodity sector

Commodity producers combine capital K_t^p , labour H_t^p and land L_t^p to produce a commodity input. It gives the capital to labour ratio:

$$\frac{k_t^p}{H_t^p} = \mu_z \left(\frac{\alpha_p \bar{w}_t^p R_t^L}{(1 - \alpha_p - \beta_p) r_t^{k,p}} \right)^{\sigma_p} \left(\frac{H_0^p}{\epsilon_{h,t} \epsilon_{hp,t} K_0^p} \right)^{\sigma_p - 1}, \quad (27)$$

and the capital to land ratio

$$\frac{k_t^p}{L_t^p} = \mu_z \left(\frac{\beta_p r_t^{k,l}}{(1 - \alpha_p - \beta_p) r_t^{k,p}} \right)^{\sigma_p} \left(\frac{L_0^p}{K_0^p} \right)^{\sigma_p - 1}, \quad (28)$$

The variables in the first order conditions can be stationarized using

$$k_{t+1}^p \equiv \frac{K_{t+1}^p}{z_t} \quad (29)$$

The relation between price and production costs is given by

$$\begin{aligned} \frac{S_t P_t^{*p}}{P_t} &= (1 - \omega_x) \left[\alpha_p \left(\frac{r_t^{k,p}}{r_t^{k,p}} \right)^{1-\sigma_p} + \beta_p \left(\frac{r_t^{k,l}}{r_t^{k,l}} \right)^{1-\sigma_p} + (1 - \alpha_p - \beta_p) \left(\frac{\bar{w}_t^p R_t^L}{\epsilon_{h,t} \epsilon_{hp,t} \bar{w}^p R^L} \right)^{1-\sigma_p} \right]^{\frac{1}{1-\sigma_p}} \\ &+ \omega_x \frac{P_t^m}{P_t} \end{aligned} \quad (30)$$

² Identical for each sector since we assumed that the import content in investment are identical across sectors

and imports by commodity producers amount to $\omega_x x_t^p$.

A.2.2 Secondary sector

Secondary good producers In the first step, cost minimization problem for the intermediate firm i in period t is given by:

$$\min_{K_{i,t}^f, H_{i,t}^f} W_t^f R_{t-1}^L H_{i,t}^f + R_t^{k,f} K_{i,t}^f + \lambda_t P_{i,t} N_{i,t}^d \quad (31)$$

where $R_t^{k,f}$ denotes the gross nominal rental rate per unit of capital services $K_{i,t}^f$, W_t^f is the nominal wage rate per unit of aggregate, homogeneous, labour $H_{i,t}^f$, R_{t-1}^L represents the gross effective nominal interest rate paid on the wage bill, the Lagrangian multiplier $\lambda_t P_{i,t}$ represents the nominal cost of producing an additional unit of the domestic good (the marginal cost) and λ_t is the real marginal cost.

The first order conditions, with respect to $H_{i,t}^f$ and $K_{i,t}^f$, for firm's i domestic input cost minimization problem are given by:

$$W_t^f R_{t-1}^L = (1 - \alpha) \lambda_t P_{i,t} N_0 \left(\frac{z_t \epsilon_{h,t} H_{i,t}^f}{H_0} \right)^{\frac{\sigma_d - 1}{\sigma_d}} \left(\frac{1}{H_{i,t}^f} \right) \quad (32)$$

$$\begin{aligned} & \times \left[\alpha \left(\frac{K_{i,t}^f}{K_0} \right)^{\frac{\sigma_d - 1}{\sigma_d}} + (1 - \alpha) \left(\frac{z_t \epsilon_{h,t} H_{i,t}^f}{H_0} \right)^{\frac{\sigma_d - 1}{\sigma_d}} \right]^{\frac{1}{\sigma_d - 1}} \\ R_t^{k,f} &= \alpha \lambda_t P_{i,t} N_0 \left(\frac{K_{i,t}^f}{K_0} \right)^{\frac{\sigma_d - 1}{\sigma_d}} \left(\frac{1}{K_{i,t}^f} \right) \\ & \times \left[\alpha \left(\frac{K_{i,t}^f}{K_0} \right)^{\frac{\sigma_d - 1}{\sigma_d}} + (1 - \alpha) \left(\frac{z_t \epsilon_{h,t} H_{i,t}^f}{H_0} \right)^{\frac{\sigma_d - 1}{\sigma_d}} \right]^{\frac{1}{\sigma_d - 1}} \end{aligned} \quad (33)$$

From those equations we can find the capital to labour ratio:

$$\frac{k_t^f}{H_t^f} = \mu_z \left(\frac{\alpha \bar{w}_t^f R_{t-1}^L}{(1 - \alpha) r_t^{k,f}} \right)^{\sigma_d} \left(\frac{H_0}{\epsilon_{h,t} K_0} \right)^{\sigma_d - 1}, \quad (34)$$

As well as the equilibrium real marginal cost of the domestic input mc_t^{nd} :

$$mc_t^{nd} \equiv \lambda_t = \left[\alpha^{\sigma_d} \left(\frac{K_0}{N_0} \right)^{1 - \sigma_d} (r_t^{k,f})^{1 - \sigma_d} + (1 - \alpha)^{\sigma_d} \left(\frac{H_0}{N_0} \right)^{1 - \sigma_d} (\bar{w}_t^f R_{t-1}^L)^{1 - \sigma_d} \right]^{\frac{1}{1 - \sigma_d}} \quad (35)$$

which, using the steady-state relationships described in the next subsection simplifies to:

$$mc_t^{nd} \equiv \lambda_t = \left[\alpha \left(\frac{r_t^{k,f}}{r^{k,f}} \right)^{1 - \sigma_d} + (1 - \alpha) \left(\frac{\bar{w}_t^f R_{t-1}^L}{\epsilon_{h,t} \bar{w}^f R^L} \right)^{1 - \sigma_d} \right]^{\frac{1}{1 - \sigma_d}} \quad (36)$$

In a second step, the firm i minimises its assembly costs

$$\min_{N_{i,t}^d, N_{i,t}^m} MC_t^{nd} N_{i,t}^d + R_{t-1}^L P_t^m N_{i,t}^m + \lambda_t' P_{i,t} Y_{i,t}^d \quad (37)$$

which yield the domestic to foreign input ratio

$$\frac{n_t^m}{n_t^d} = \left(\frac{\omega_n}{1 - \omega_n} \frac{mc_t^{nd}}{R_{t-1}^L P_t^m / P_t} \right)^{\sigma_n} \left(\frac{N_0^d}{N_0^m} \right)^{\sigma_n - 1}, \quad (38)$$

as well as the marginal cost mc_t

$$mc_t \equiv \lambda_t' = \frac{1}{\lambda_d} \times \left[\omega_n \left(\frac{R_{t-1}^L P_t^m / P_t}{R^L} \right)^{1 - \sigma_n} + (1 - \omega_n) \left(\frac{mc_t^{nd}}{mc^{nd}} \right)^{1 - \sigma_n} \right]^{\frac{1}{1 - \sigma_n}} \quad (39)$$

The variables in the first order conditions have been stationarized using

$$k_{t+1}^f \equiv \frac{K_{t+1}^f}{z_t}; \quad n_t^f \equiv \frac{N_t^f}{z_t} \text{ and } mc_t^{nd} \equiv \frac{MC_t^{nd}}{P_t} \quad (40)$$

Domestic Distributors The profit maximization problem for the final good distributor gives the following first order condition:

$$\frac{Y_{i,t}^f}{Y_t^f} = \left(\frac{P_t}{P_{i,t}} \right)^{\frac{\lambda_{d,t}}{\lambda_{d,t} - 1}} \quad (41)$$

where P_t is the price for the homogeneous final good and $P_{i,t}$ is the input price for the intermediary good i , taken as given by the final good firm. The price index P_t is computed is given by:

$$P_t = \left[\int_0^1 P_i^{\frac{1}{1 - \lambda_{d,t}}} di \right]^{(1 - \lambda_{d,t})} \quad (42)$$

The optimization problem faced by the intermediate distributor i when setting its price at time t taking aggregator's demand as given reads:

$$\max_{P_t^{new}} E_t \sum_{s=0}^{\infty} (\beta_E \xi_d)^s v_{t+s}^e [((\pi_t \pi_{t+1} \dots \pi_{t+s-1})^{\kappa_d} (\bar{\pi}_{t+1}^c \bar{\pi}_{t+2}^c \dots \bar{\pi}_{t+s}^c)^{1 - \kappa_d} P_t^{new}) Y_{i,t+s}^f - MC_{i,t+s} Y_{i,t+s}^f], \quad (43)$$

where $(\beta_E \xi_d)^s v_{t+s}^e$ is a stochastic discount factor, v_{t+s}^e the marginal utility of entrepreneurs' nominal income in period $t + s$ and $MC_{i,t}$ is the firm's nominal marginal cost. Using (41) the first order condition for this optimization problem can be written as:

$$\begin{aligned} E_t \sum_{s=0}^{\infty} (\beta_E \xi_d)^s v_{t+s}^e & \left(\frac{\left(\frac{P_{t+s-1}}{P_{t-1}} \right)^{\kappa_d} (\bar{\pi}_{t+1}^c \bar{\pi}_{t+2}^c \dots \bar{\pi}_{t+s}^c)^{1-\kappa_d}}{\left(\frac{P_{t+s}}{P_t} \right)} \right)^{-\frac{\lambda_{d,t+s}}{\lambda_{d,t+s}-1}} Y_{t+s}^d P_{t+s} \times \\ & \left[\frac{\left(\frac{P_{t+s-1}}{P_{t-1}} \right)^{\kappa_d} (\bar{\pi}_{t+1}^c \bar{\pi}_{t+2}^c \dots \bar{\pi}_{t+s}^c)^{1-\kappa_d}}{\left(\frac{P_{t+s}}{P_t} \right)} \frac{P_t^{new}}{P_t} - \frac{\lambda_{d,t} MC_{\iota,t+s}}{P_{t+s}} \right] = 0. \end{aligned} \quad (44)$$

which gives the price Phillips-Curve.

Importing distributors Optimisation in the importing firms price setting problem is similar to the domestic good price setting problem presented above. The difference is that importing firms face the following marginal cost to sale price ratio

$$\frac{S_t MC_t^*}{P_t^m} \quad (45)$$

For details we refer to [Adolfson et al. \(2007\)](#)³.

Exporting distributors Optimisation in the exporting firms price setting problem is similar to the domestic good price setting problem presented above. The difference is that exporting firms face the following marginal cost to sale price ratio

$$\frac{(1 - \omega_x) MC_t + \omega_x P_t^m}{S_t P_t^x} \quad (46)$$

For further details on the Calvo export price setting problem we refer to [Adolfson et al. \(2007\)](#)⁴. Foreign demand is give by

$$x_t^* = X^* \left(\frac{\nu c_t^* + (1 - \nu) i_t^*}{\nu c^* + (1 - \nu) i^*} \right) \varepsilon_{x,t}, \quad (47)$$

where and $1 - \nu$ is the share of investment goods in foreign trade and domestic exports are given by

$$x_t^f = \left(\frac{P_t^x}{P_t^*} \right)^{-\eta_f} x_t^* \quad (48)$$

Due to the inclusion of an import content of exports (with a Leontief technology) we have that imports of exporting firms amount to $\omega_x x_t^f$.

³ The only difference with [Adolfson et al. \(2007\)](#) is that the importing firm consider MC_t^* instead of P_t^* for its nominal costs.

⁴ The only difference with [Adolfson et al. \(2007\)](#) is that the exporting firm consider $(1 - \omega_x) MC_t + \omega_x P_t^m$ instead of P_t for its nominal costs.

A.3 Financial sector

The domestic bank determines the lending rate $R_t^{L,d}$ and charge an external financing premium over the risk free rate to finance monitoring costs by setting

$$R_t^{L,d} = S \left(\frac{b_t^e}{v_t} \right) R_t + \varepsilon_t^b + \omega_k \varepsilon_t^{b*} \quad (49)$$

where $v_t = y_t^f + \frac{S_t P_t^{*p}}{P_t} x_t^p$ is the real (stationary) value of collateral.

A.4 Public authorities

The Government and Central Bank follow simple rules already presented in their stationary form in the model section.

A.5 Closing market conditions and definitions

In equilibrium the final goods market, the loan market and the foreign bond market have to clear. The aggregate resource constraint has to satisfy the following condition on the use of the domestic good:

$$c_t^d + i_t^d + g_t + (1 - \omega_x) x_t^f \leq y_t^f - a(u_t) \frac{\bar{k}_t}{\mu_z} \quad (50)$$

Foreign assets evolves according to:

$$a_t^* = R_{t-1}^* \Phi \left(a_{t-1}, \tilde{\phi}_{t-1}^a \right) \frac{a_{t-1} \Delta S_t}{\pi_t \mu_z} + \frac{S_t P_t^x}{P_t} x_t^f + \frac{S_t P_t^{*p}}{P_t} x_t^p \quad (51)$$

$$- \frac{P_t^m}{P_t} \left(c_t^m + i_t^m + n_t^m + \omega_x (x_t^f + x_t^p) \right) \quad (52)$$

And the GDP identity is given by

$$y_t = c_t + i_t + g_t + x_t - m_t \quad (53)$$

where $c_t = c_t^s + c_t^e + c_t^r$, $i_t = i_t^p + i_t^f$, $x_t = x_t^f + x_t^p$ and $m_t = c_t^m + i_t^m + n_t^m + \omega_x x_t$

A.6 Foreign Economy

The techniques used to derive foreign economy equations are similar to those presented in the domestic block and are not reported here.

Appendix B: model's steady state

Here we provide the details on the computation of steady-state for the domestic economy. First we fix some values reflecting some freedom in the choice of units:

$$\begin{aligned} y^f &= Y_0^f = 1 \\ P_0 &= P_0^* = 1 \\ z_0 &= z_0^* = 1 \\ h &= 0.3 \end{aligned}$$

where Y_0^f , P_0 and z_0 are free choice of units and $h_j = 0.3$ ensures that agents devote on average 30% of their time to labour activities and just imposes to calibrate A_L accordingly. It implies that total hours worked by savers and rule of thumb consumers is given by $H = H^r + H^s$ and that the time they spend working in each sectors is given by $H^p = H_0^p = \omega_h H$ and $H^f = H_0^f = (1 - \omega_h)H$.

The primary commodity sector's share in GDP is calibrated to ω_p to match its empirical counterpart. it implies that

$$\begin{aligned} Y &= \frac{Y^f}{1 - \omega_p} \\ Y^p &= Y_0^p = \omega_p Y \end{aligned}$$

We also assume that the trend in both labour productivity and prices are the same in the domestic and foreign economies such that

$$\begin{aligned} \pi &= \pi^* = \bar{\pi} \\ dS &= 1 \end{aligned}$$

where $\bar{\pi}$ is calibrated to match the trend in inflation in the domestic economy. As a consequence, all inflations rates are equal to $\bar{\pi}$. By carefully calibrating mark-ups for each distributors we can equalize all relative prices $\frac{P^i}{P^j}$ to one at steady-state for simplicity.

Turning to patient households FOCs, we can calibrate the discount factor for savers β_S using

$$\beta_S = \frac{\pi \mu_z}{R - \tau^k (R - 1)}$$

where μ_z and R are the average growth rate and risk-free interest rate in the domestic economy.

With entrepreneurs FOC we can get

$$\begin{aligned} \beta_E &= \frac{\pi \mu_z}{R^L - \tau^k (R^L - 1)} \\ p_{k'} &= \frac{P^k}{P} = \frac{P^i}{P} = 1 \\ r^k &= \frac{p_{k'}(\mu_z - (1 - \delta)\beta_E)}{(1 - \tau^k)\beta_E} \end{aligned}$$

where $R^L - R$ is calibrated to match the average spread between risk free and lending rates.

Turning to firms, the firms marginal costs are given by

$$\begin{aligned} mc &= 1/\lambda_d \\ mc^m &= 1/\lambda_d \\ mc^x &= (1 - \omega_x)mc + \omega_x \end{aligned}$$

which implies that $\lambda_m = 1/mc^m$ and $\lambda_x = 1/mc^x$. Of course, in the perfectly competitive producing sectors, real marginal costs are given by

$$\begin{aligned} mc^p &= 1 \\ mn^{nd} &= 1 \end{aligned}$$

In addition, the use of a normalised CES as in [Cantore and Levine \(2012\)](#) in the final good sector allows us to write

$$\underbrace{y^f}_1 = \underbrace{R^L \frac{P^n}{P} n^m}_{\omega_n} + \underbrace{\frac{MC^{nd}}{P} n^d}_{1-\omega_n}$$

Such that

$$\begin{aligned} n^d &= 1 - \omega_n = N_0^d \\ n^m &= \omega_n / R^L = N_0^m \end{aligned}$$

As well as

$$\begin{aligned} mc^{nd} n^d &= \frac{r^k k^f}{\mu_z} + \bar{w}^f H^f R^L \\ \alpha &= \frac{r^k k^f}{\mu_z mc^{nd} n^d} \\ (1 - \alpha) &= \frac{\bar{w}^f H^f}{mc^{nd} n^d} \end{aligned}$$

such that

$$\begin{aligned} k^f &= \frac{\alpha \mu_z mc^{nd} n^d}{r^k} = \frac{\alpha (1 - \omega_n) \mu_z}{r^k} \\ \bar{w}^f &= \frac{(1 - \alpha) mc^{nd} n^d}{H^f R^L} = \frac{(1 - \alpha) (1 - \omega_n)}{H^f R^L} \end{aligned}$$

and wages are equal across sectors at steady-state so $\bar{w} = \bar{w}^p = \bar{w}^f$. It also implies that we can find the value of investment

$$i^f = \left(1 - \frac{1 - \delta}{\mu_z}\right) k^f$$

Turning to commodity producers, we know \bar{w}^p and y^p . Using once again a Normalised CES implies that

$$mc^p y^p = r^{k,l} L^p + R^L \bar{w} H^p + \frac{r^k k^p}{\mu_z}$$

with the following capital and labour income shares:

$$\begin{aligned} \frac{r^k k^p}{\mu_z} &= \alpha_p mc^p y^p \\ R^L \bar{w} H^p &= (1 - \alpha_p - \beta_p) mc^p y^p \end{aligned}$$

It implies that

$$k^p = \frac{\mu_z \alpha_p mc^p y^p}{r^k}$$

and that

$$\beta_p = 1 - \alpha_p - \frac{R^L \bar{w} H^p}{mn^p y^p}$$

where β_p is fixed such that the labour income share $\frac{R^L \bar{w} H^p}{mn^p y^p}$ matches our assumption on hours worked in the primary sector. Therefore,

$$\begin{aligned} i^p &= \left(1 - \frac{1 - \delta}{\mu_z}\right) k^p \\ y &= y^d + y^p - n^m \end{aligned}$$

and

$$\begin{aligned} i &= i^f + i^p \\ i^m &= \omega_i i \\ i^d &= (1 - \omega_i) i \end{aligned}$$

The aggregate resource constraint evaluated at steady state reads

$$y^f - g = c^d + i^d + (1 - \omega_x) x^f$$

Plugging, steady state domestic consumption values from households yields

$$y^f - g = (1 - \omega_c) c + i^d + (1 - \omega_x) x^f$$

Assuming we can calibrate the net foreign asset position, the assets accumulation rule gives

$$c^m + i^m + n^m + \omega_x (x^p + x^f) = x^p + x^f + \left(\frac{R}{\pi \mu_z} - 1\right) a$$

where $y^p = (1 - \omega_x)x^p$. Knowing steady state value of imported consumption we have,

$$\omega_c c + i^m + n^m = y^p + (1 - \omega_x)x^f + \left(\frac{R}{\pi\mu_z} - 1\right) a$$

We now have two equations with only x^f and c unknown. Solving yields

$$c = y^f - (i^m + i^d + n^m + g) + y^p + \left(\frac{R}{\pi\mu_z} - 1\right) a$$

such that $c^m = \omega_c c$, $c^d = (1 - \omega_c)c$ and

$$x^f = \frac{1}{1 - \omega_x}(y^f - g - c^d - i^d)$$

We have the value of aggregate consumption $c = c^s + c^e + c^r$. The consumption of rule of thumbs households is given by $c^r = \frac{1-\tau_y}{(1+\tau_w)(1+\tau_c)}\bar{w}H^r + tr^r$ where tr^r can be set in order to attain any objective on c^r including $c^r = c/3$. We use the same strategy for entrepreneurs by calibrating tr^e to get $c^e = c/3$.

Finally, production functions written in normalised forms will satisfy $y^f = Y_0^f$ and $y^p = Y_0^p$ if $K_0^p = k^p/\mu_z$ and $K_0^f = k^f/\mu_z$.

Appendix C: Observation Equations

We have a set of 13 domestic and 9 foreign observed variables linked to the model:

$$\begin{pmatrix}
 100\log(GDP_t/GDP_{t-4}) \\
 100\log(CONS_t/CONS_{t-4}) \\
 100\log(INV_t/INV_{t-4}) \\
 100\log(EXP_t/EXP_{t-4}) \\
 100\log(COM_t/COM_{t-4}) \\
 100\log(IMP_t/IMP_{t-4}) \\
 100\log(EMP_t/EMP_{t-4}) \\
 REPO_t \\
 100\log(CPI_t/CPI_{t-4}) \\
 100\log(MPI_t/MPI_{t-4}) \\
 -100\log(NEER_t/NEER_{t-1}) \\
 SPREAD_t \\
 100\log\left(\frac{LABCOMP_t/CPI_t}{LABCOMP_{t-4}/CPI_{t-4}}\right) \\
 - - - \\
 100\log(GDP^*/GDP^*_{t-4}) \\
 100\log(CONS^*/CONS^*_{t-4}) \\
 100\log(INV^*/INV^*_{t-4}) \\
 100\log(H^*_t/H^*_{t-4}) \\
 FFR_t \\
 100\log(CPI^*_t/CPI^*_{t-4}) \\
 100\log(WAGE^*_t/WAGE^*_{t-4}) \\
 SPREAD^*_t \\
 100\log\left(\frac{CP^*_t/CP^*_{t-4}}{CP^*_{t-4}/CP^*_{t-4}}\right)
 \end{pmatrix}
 =
 \begin{pmatrix}
 \bar{\gamma}^y \\
 \bar{\gamma}^c \\
 \bar{\gamma}^i \\
 \bar{\gamma}^x \\
 \bar{\gamma}^p \\
 \bar{\gamma}^m \\
 \bar{\gamma}^e \\
 \bar{\gamma}^r \\
 \bar{\gamma}^\pi \\
 \bar{\gamma}^{\pi^m} \\
 \bar{\gamma}^{\Delta S} \\
 \bar{\gamma}^s \\
 \bar{\gamma}^w \\
 - \\
 \bar{\gamma}^{y^*} \\
 \bar{\gamma}^{c^*} \\
 \bar{\gamma}^{i^*} \\
 \bar{\gamma}^{h^*} \\
 \bar{\gamma}^{r^*} \\
 \bar{\gamma}^{\pi^*} \\
 \bar{\gamma}^{\pi^{w*}} \\
 \bar{\gamma}^{s^*} \\
 \bar{\gamma}^{cp^*}
 \end{pmatrix}
 +
 \begin{pmatrix}
 100\log(y_t/y_{t-4}) \\
 100\log(c_t/c_{t-4}) \\
 100\log(i_t/i_{t-4}) \\
 100\log(x_t/x_{t-4}) \\
 100\log(y^p_t/y^p_{t-4}) \\
 100\log(m_t/m_{t-4}) \\
 100\log(E_t/E_{t-4}) \\
 400R_t \\
 100\log(\pi^c_t\pi^c_{t-1}\pi^c_{t-2}\pi^c_{t-3}) \\
 100\log(\pi^m_t\pi^m_{t-1}\pi^m_{t-2}\pi^m_{t-3}) \\
 100\log(\Delta S_t) \\
 400\omega_s(R^L_t - R_t) \\
 100\log\left(\frac{\bar{w}_t H_t P_t / P^c_t}{\bar{w}_{t-4} H_{t-4} P_{t-4} / P^c_{t-4}}\right) \\
 - - - \\
 100\log(y^*_t/y^*_{t-4}) \\
 100\log(c^*_t/c^*_{t-4}) \\
 100\log(i^*_t/i^*_{t-4}) \\
 100\log(H^*_t/H^*_{t-4}) \\
 400R^*_t \\
 100\log(\pi^*_{t-1}\pi^*_{t-2}\pi^*_{t-3}) \\
 100\log(\pi^{w*}_{t-1}\pi^{w*}_{t-2}\pi^{w*}_{t-3}) \\
 400\omega^*_s(R^{L*}_t - R^*_t) \\
 100\log(\gamma^{p*}_t/\gamma^{p*}_{t-4})
 \end{pmatrix}
 +
 \begin{pmatrix}
 \epsilon^y_t \\
 \epsilon^c_t \\
 \epsilon^i_t \\
 \epsilon^x_t \\
 \epsilon^p_t \\
 \epsilon^m_t \\
 \epsilon^e_t \\
 \epsilon^r_t \\
 \epsilon^\pi_t \\
 \epsilon^{\pi^m}_t \\
 \epsilon^{\Delta S}_t \\
 \epsilon^s_t \\
 \epsilon^w_t \\
 - \\
 \epsilon^{y^*}_t \\
 \epsilon^{c^*}_t \\
 \epsilon^{i^*}_t \\
 \epsilon^{h^*}_t \\
 \epsilon^{r^*}_t \\
 \epsilon^{\pi^*}_t \\
 \epsilon^{\pi^{w*}}_t \\
 \epsilon^{s^*}_t \\
 \epsilon^{cp^*}_t
 \end{pmatrix}$$

where $\bar{\gamma}$ are constants calibrated at the corresponding observed series mean. This departs from the traditional view that the trend in real variables should be identical. However, considering that trade shares have been growing in South Africa over the estimation period (starting after the end of the apartheid), and that growth rates were higher in South Africa than in the US, we decided to allow for different means in the observation equations. Similar arguments hold for average inflation and interest rates. Measurement errors ϵ are calibrated to relatively small values for all variables with the exception of exports and imports. Since we only estimate one shock specific to trade volumes, estimated measurement errors are necessary to match exports and imports. The parameter ω_s and ω^*_s represent the share of the external financing costs captured by movement in the domestic and foreign spread. We justify it by the presence of other costs such as search costs that presumably move in the same direction and by the fact that short term risk premiums are more volatile than their long run counterparts. In the model, we have used hours worked while in the data only employment is available. In order to capture labour hoarding we define employment following [Adolfson et al. \(2007\)](#) as

$$E_t = \frac{1}{1+\beta} E_{t-1} + \frac{\beta}{1+\beta} E_{t+1} + \frac{(1-\xi_e)(1-\beta\xi_e)}{(1+\beta)\xi_e} (H_t - E_t) \quad (54)$$

where $1 - \xi_e$ is the probability of a firm to be allowed to readjust employment.

Appendix D: Data transformations

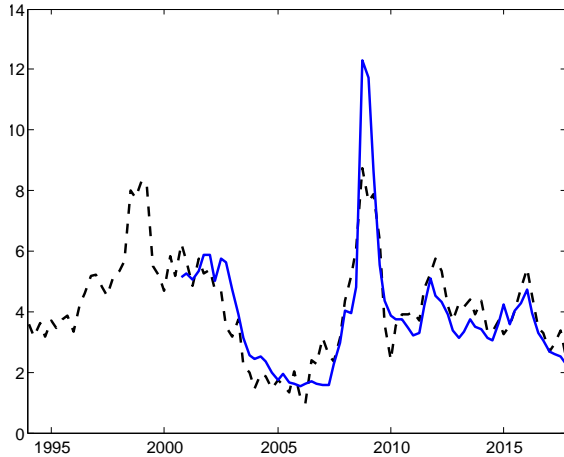
Some specific data transformation are detailed here. Data sources and transformations applied to other variables in Table 1.

South African spread proxy We proxy the South African spread using the predicted values obtained from regressing an emerging market spread index on South African variable such as the number of insolvencies, the yield on EKSOM bonds, the spread between domestic and US 10 years government bonds yields, the OECD-MEI manufacturing business confidence indicator and the MSCI mid and large cap equity return index. Figure 1(a) shows the emerging market spread index together with the fitted values from its regression on South Africa variables. The regression is performed on monthly data over the 1999M1 to 2018M7. Predicted values are computed based on this relation for the 1994M1 to 2017M12 period. We average over this monthly indicator to build our quarterly spread proxy.

South African commodity export proxy Commodity exports are proxied by total mining sales from the Stat SA database divided by the export price index from the SARB database. Since about 70% of mining production is exported, this measure gives a good proxy of mining exports. For illustrative purposes, it is compared to the growth rate in real total exports in Figure 1(b) below.

Figure 1: Data proxies

(a) Spread proxy (dashed black) compared to emerging market spread index (blue). Rates annualised.



(b) Mining export proxy in volumes (black) compared to total exports volumes (blue). YoY growth rates.

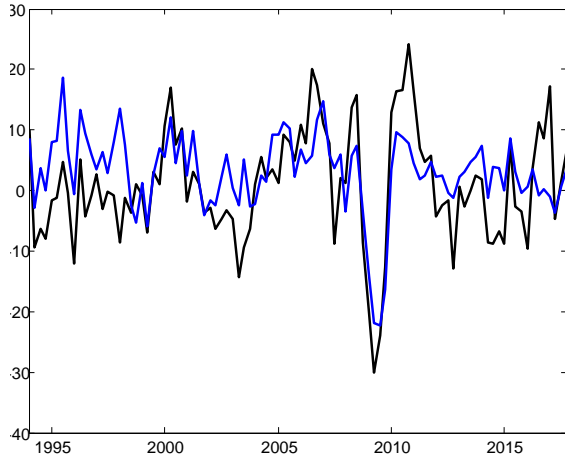


Table 1: Data sources and transformations

South African Data	Source	Transformation
GDP	Stat SA	GDP. Constant price. YoY.
Employment	SARB	Employment in the private sector (index). YoY.
Consumption	Stat SA	Final Consumption Expenditure, Households. Constant price. YoY.
Investment	Stat SA	Gross Fixed Capital Formation. Constant price. YoY.
Exports	Stat SA	Exports. Constant prices. YoY.
Imports	Stat SA	Imports. Constant prices. YoY.
Mining exports	Stat SA	total value of mineral sales deflated by export prices. YoY.
CPI	Stat SA	Consumer Price Index, Urban Areas, All Items. YoY.
Import price index	SARB	Import price index for goods and services. YoY.
Labor compensation	Stat SA	Compensation of Employees. Deflated by CPI. YoY.
Risk-free rate	SARB	Repo Rate. Annual rate in level.
Spread	Various sources	Authors computations. See appendix D.
NEER	BIS	Nominal broad effective exchange rate. QoQ.
US data	Source	Transformation
US GDP	OECD-MEI	GDP. Constant price. YoY
US Consumption	OECD-MEI	Private Final Consumption Expenditure. Constant price. YoY.
US Investment	OECD-MEI	Gross Fixed Capital Formation. Constant price. YoY.
US Hours	US Burea of labor statistics	Hours worked, Production & Non-Supervisory Employees, Total Private. YoY.
US CPI	US Burea of labor statistics	Consumer Price Index, All Urban Consumers, U.S. City Average, All Items Less Shelter. YoY.
US Wage	US Burea of labor statistics	Nonfarm Business, Hourly Compensation. YoY.
US Risk-free rate	Wu and Xia (2016) and Fred	Effective Fed Fund rate replaced by its shadow rate over the 2009Q3 to 2015Q4 period.
US Spread	Moody's	5 year BAA corporate yield minus 5 year government bond yield. Annual rates in levels.
World data	Source	Transformation
Commodity Price	World Bank	Commodity prices deflated with US CPI. Average of coal, aluminium, platinum and silver. YoY.
G7 data	Source	Transformation
G7 GDP	OECD-MEI	GDP. G7 average. Constant price. YoY.
G7 Consumption	OECD-MEI	Private Final Consumption Expenditure. G7 average. Constant price. YoY.
G7 Investment	OECD-MEI	Gross Fixed Capital Formation. G7 average. Constant price. YoY.
G7 PPI	OECD-MEI	PPI (manufacturing). G7 average. YoY.
G7 Wage	OECD-MEI	Hourly earnings. G7 average. YoY.

YoY = growth rate same quarter of the previous year. QoQ = quarter on quarter growth rate.

Appendix E: Calibration of other parameters

Some parameters of the domestic bloc are calibrated in order to match different key steady state targets. The values are presented in the paper. The parameter μ_z is defined by the average growth rate in South-Africa. The Central bank inflation target $\bar{\pi}$ is set based on the observed average inflation rate. The parameter δ is calibrated using data on the investment to GDP ratio. The capital income share in the manufacturing sector α is set to 0.3. We set h to 0.3 to ensure that agents devote 30% of their time to labour at steady state.

The inverse of inter-temporal substitution elasticity σ_c and the inverse of labour supply elasticity σ_l are calibrated to 1 and 2, respectively. We set $\lambda_d = \lambda_m = \lambda_x = 1.25$ and $\lambda_w = 1.1$: firms in the manufacturing sector and households in the labour market enjoy monopolistic power allowing them to impose 25% and 10% mark-up at steady state, respectively.

Government consumption to GDP is calibrated to 20%. Capital gain tax rate τ_k is calibrated to 0.2; pay-roll tax τ_w to 0.05; labour income tax τ_y to 0.03 and value added tax τ_c to 0.14. We also calibrated the size of the government consumption shock to match the volatility of the empirical series. Foreign debt stock to quarterly GDP is calibrated to 0.8 (20% of annual GDP). The country risk premium elasticity to foreign asset position ϕ_a is set to 0.0001. This sufficiently low value ensures that the model is stationary but does not influence too much its dynamics at business cycle frequencies.

Some parameters traditionally estimated are calibrated to circumvent identification issues. The final good, import and export indexation parameters are set to 0.1 (they would be estimated to a very low value). The labour mobility parameter is set to 1 as in [Horvath \(2000\)](#) and [Dagher et al. \(2010\)](#).

Most other calibrated parameters in the foreign economy are set at their domestic counterparts' values with the exception of indexation in the price Phillips Curves κ^* is set to 0.2.

Appendix F: Distribution of estimated parameters and convergence diagnostic

Figure 2: Prior and Posterior distributions (foreign and SOE shocks std)

Prior distributions in grey, posterior distributions in blue, posterior modes in green. Posterior distributions plotted from 2 MCMC chains of 200 000 draws burning the first 100 000 draws.

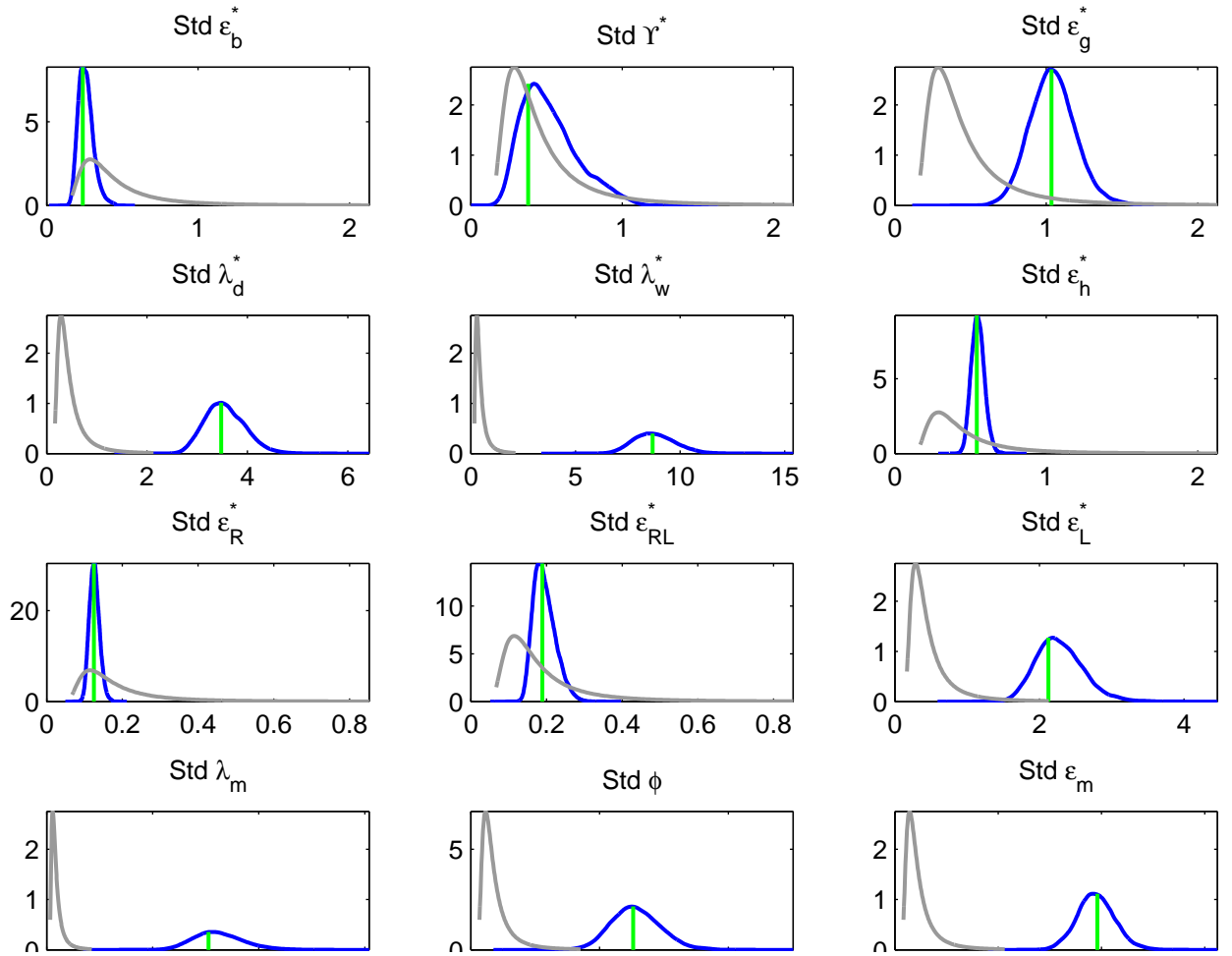


Figure 3: Prior and Posterior distributions (domestic shocks std)

Prior distributions in grey, posterior distributions in blue, posterior modes in green. Posterior distributions plotted from 2 MCMC chains of 200 000 draws burning the first 100 000 draws.

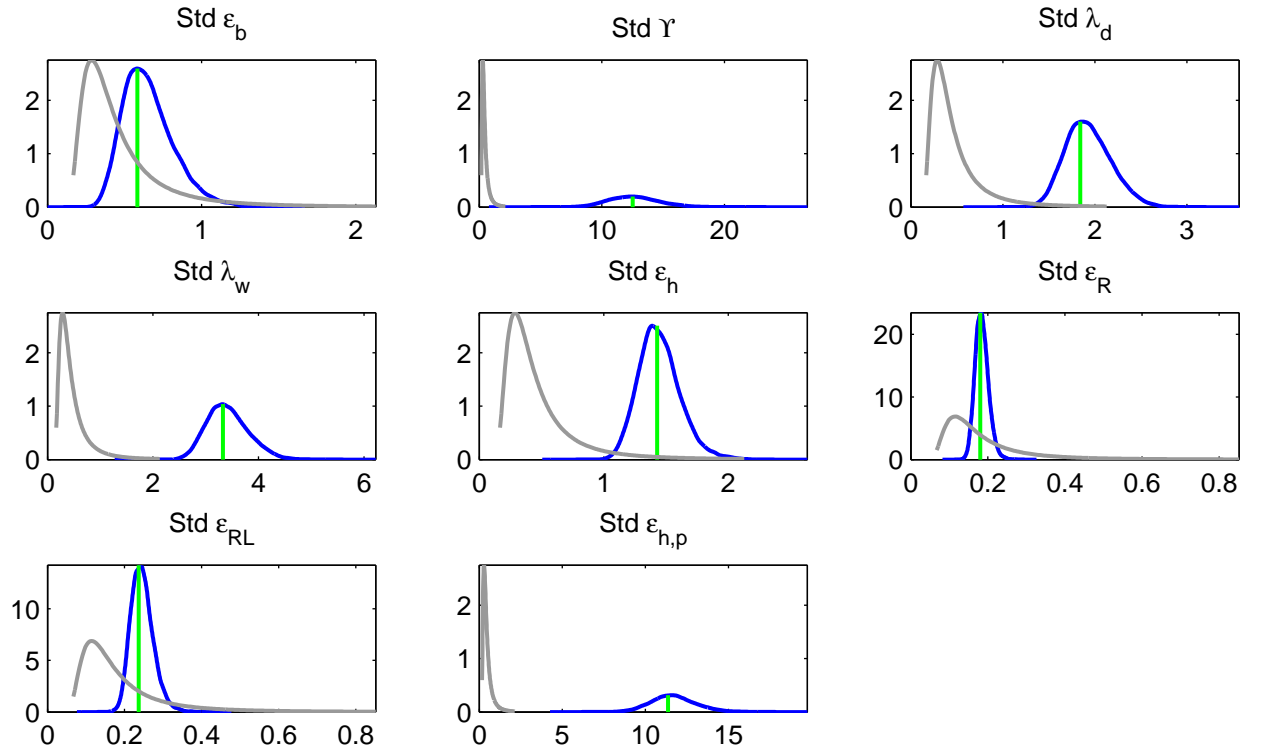


Figure 4: Prior and Posterior distributions (foreign and SOE shocks persistence)

Prior distributions in grey, posterior distributions in blue, posterior modes in green. Posterior distributions plotted from 2 MCMC chains of 200 000 draws burning the first 100 000 draws.

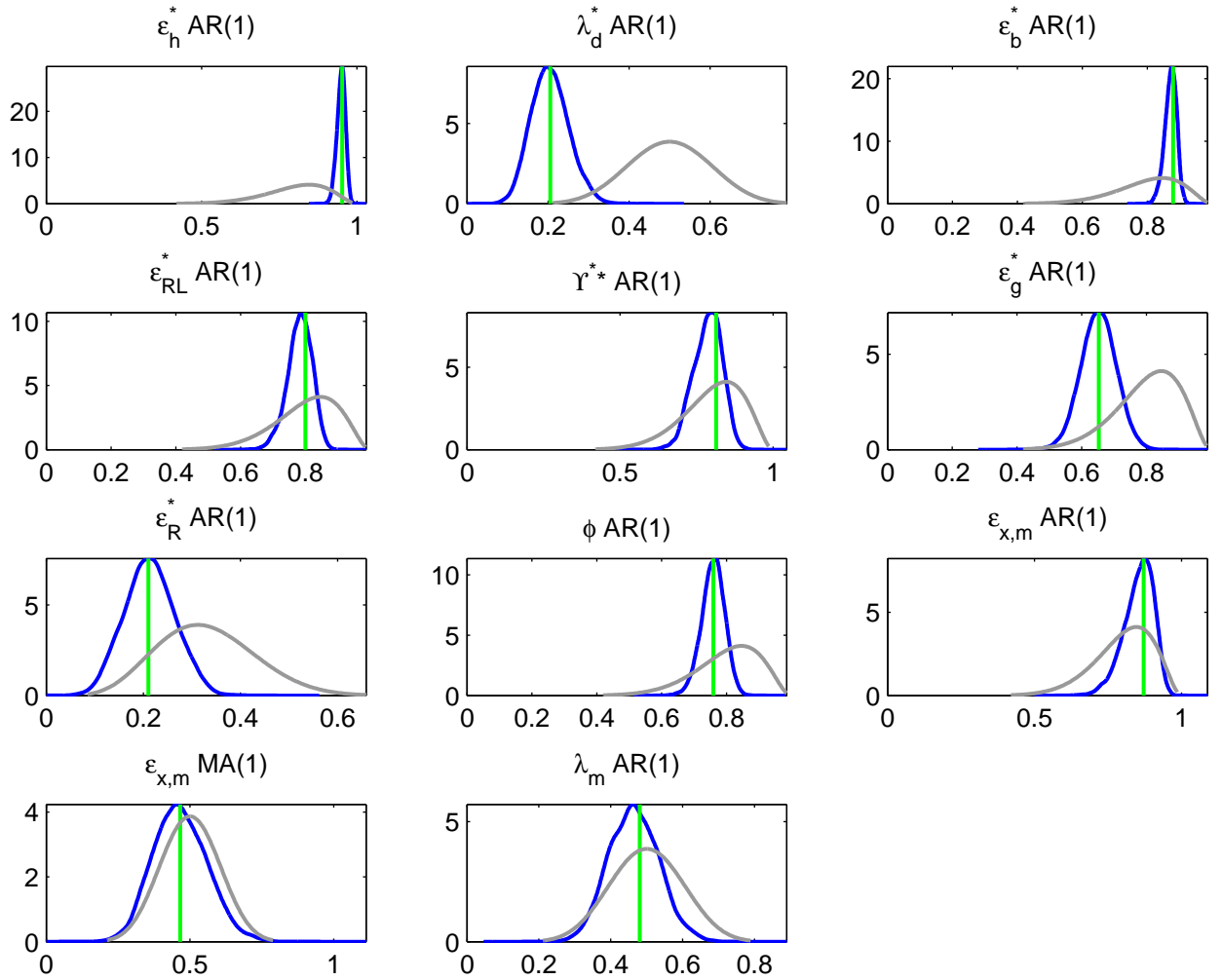


Figure 5: Prior and Posterior distributions (domestic shocks persistence)

Prior distributions in grey, posterior distributions in blue, posterior modes in green. Posterior distributions plotted from 2 MCMC chains of 200 000 draws burning the first 100 000 draws.

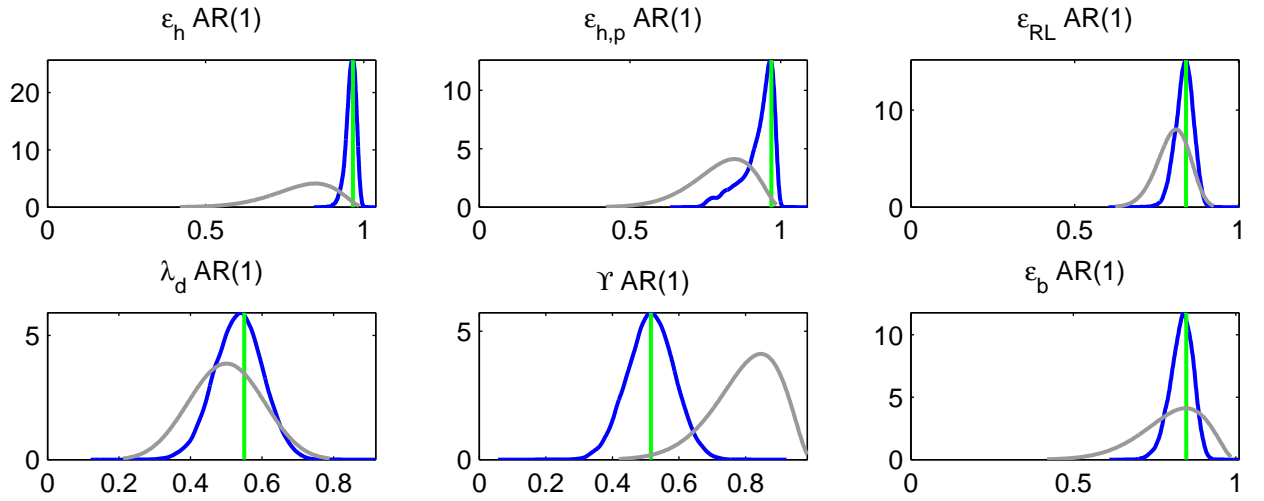


Figure 6: Prior and Posterior distributions (foreign parameters)

Prior distributions in grey, posterior distributions in blue, posterior modes in green. Posterior distributions plotted from 2 MCMC chains of 200 000 draws burning the first 100 000 draws.

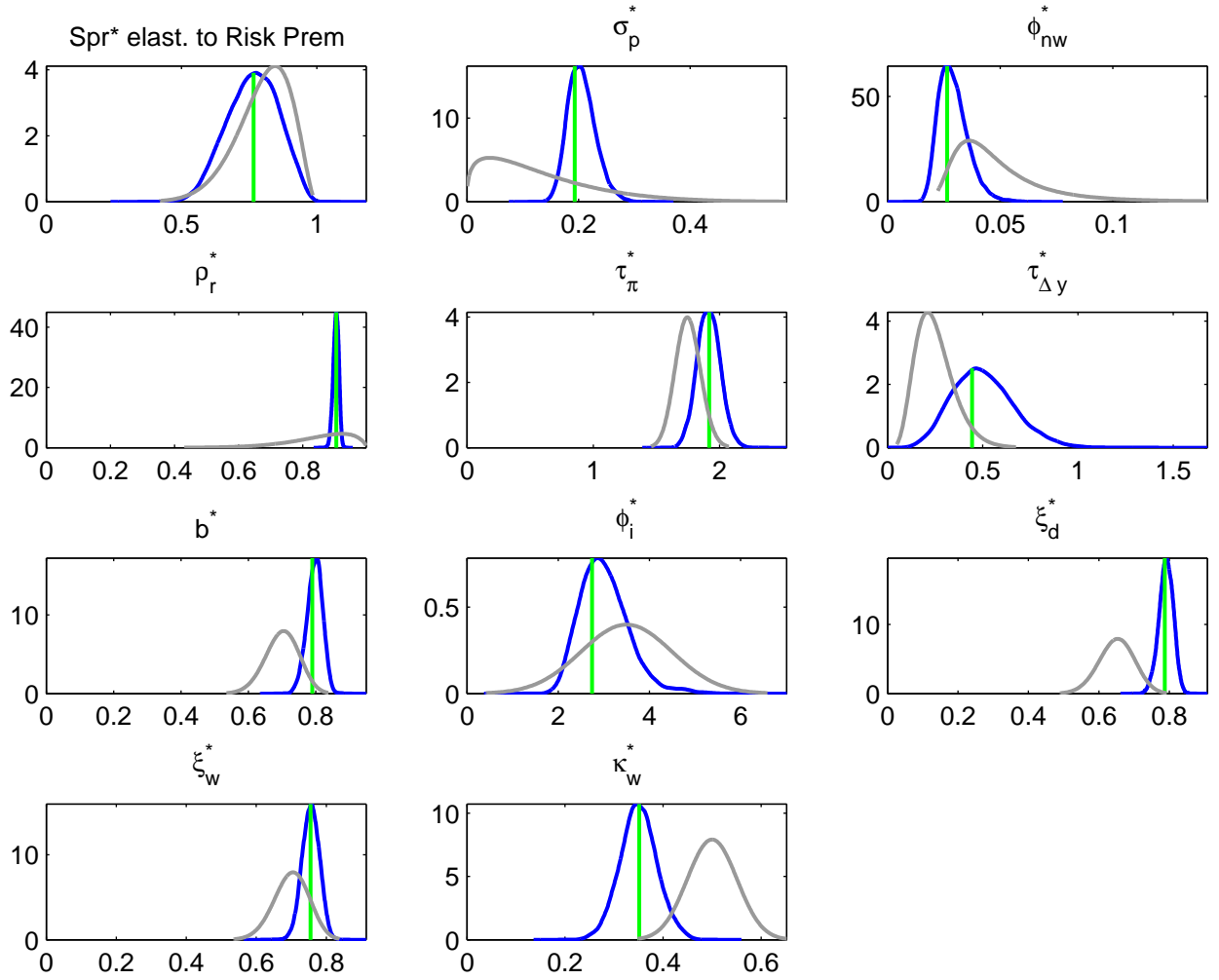


Figure 7: Prior and Posterior distributions (domestic parameters)

Prior distributions in grey, posterior distributions in blue, posterior modes in green. Posterior distributions plotted from 2 MCMC chains of 200 000 draws burning the first 100 000 draws.

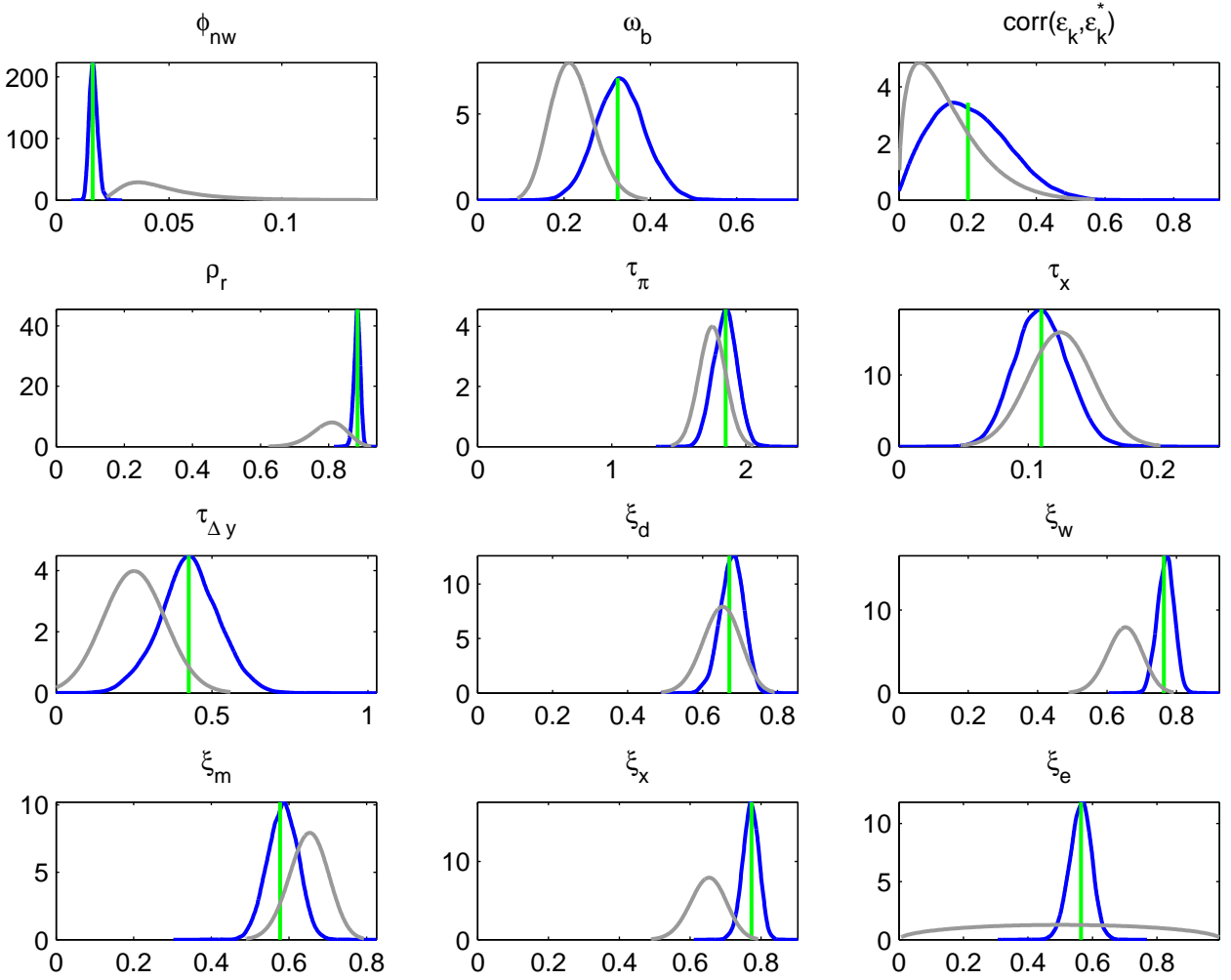


Figure 8: Prior and Posterior distributions (domestic parameters)

Prior distributions in grey, posterior distributions in blue, posterior modes in green. Posterior distributions plotted from 2 MCMC chains of 200 000 draws burning the first 100 000 draws.

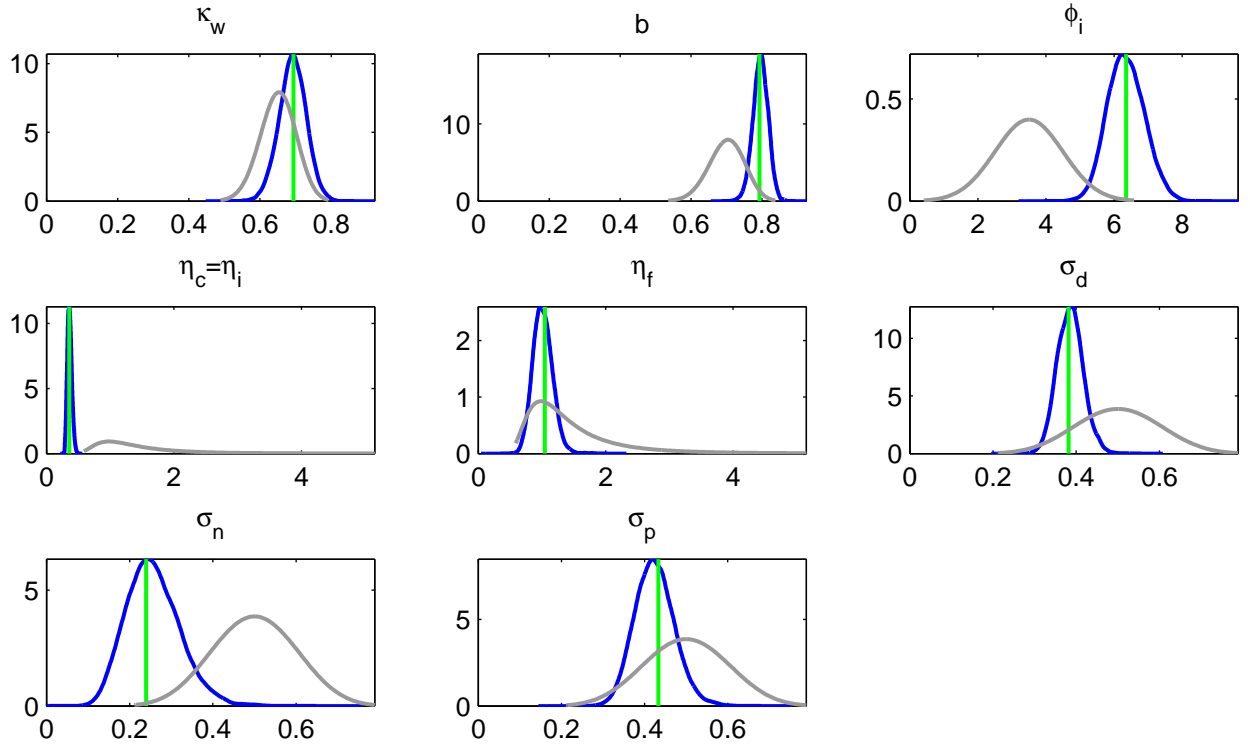
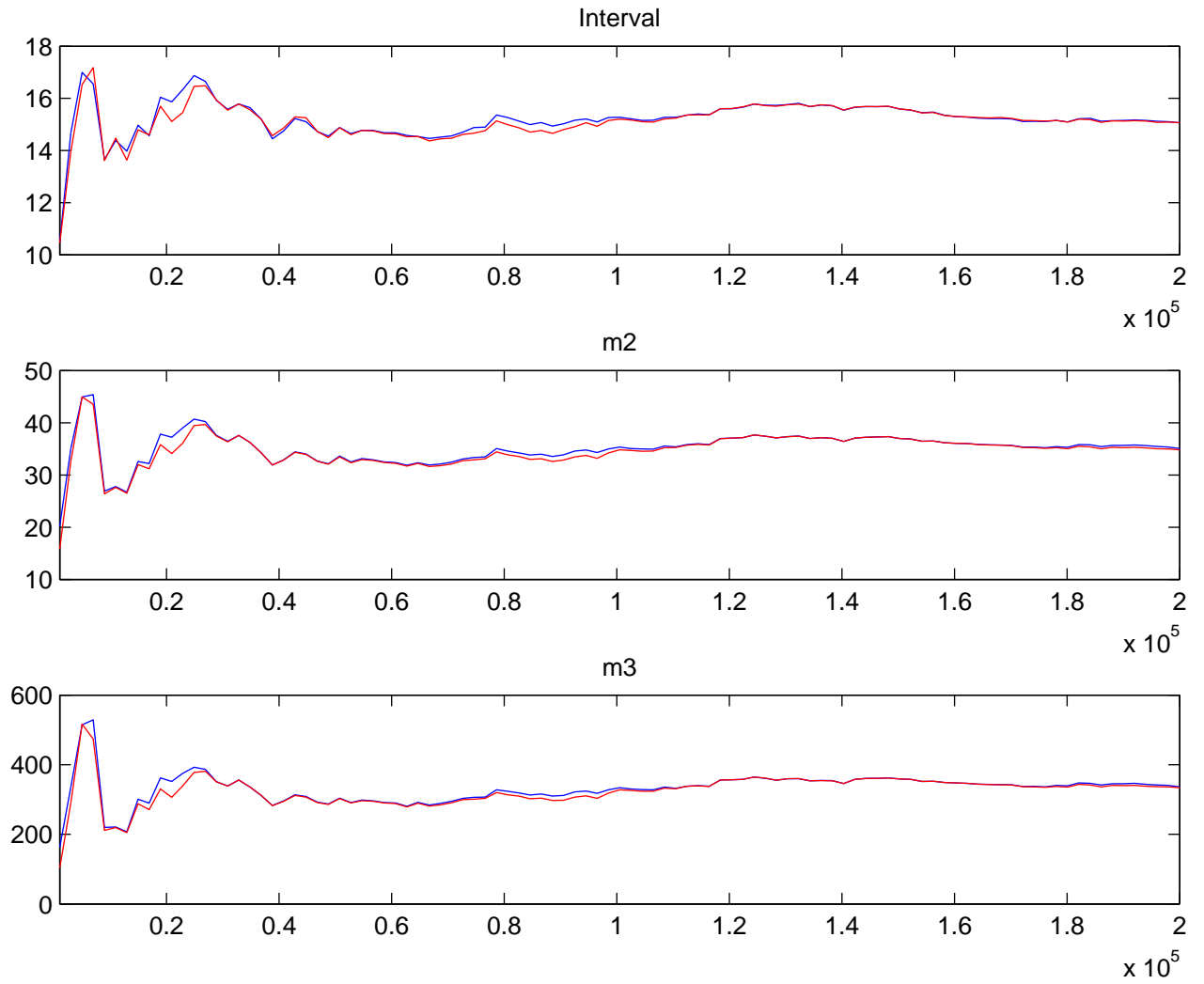


Figure 9: Convergence Diagnostic

Brooks and Gelman (1998) Convergence Diagnostic on the mean, variance ($m2$) and skewness ($m3$) based on 2 chains of 200 000 draws.



Appendix G: IRFs

Figure 10: IRFs - Foreign Commodity supply shock

Note: Variables expressed in percentage deviation from steady-state, inflation, spread and interest rates annualized. Horizon in quarters. Baseline model with SA variables in black and US variables in grey and 90% confidence bands.

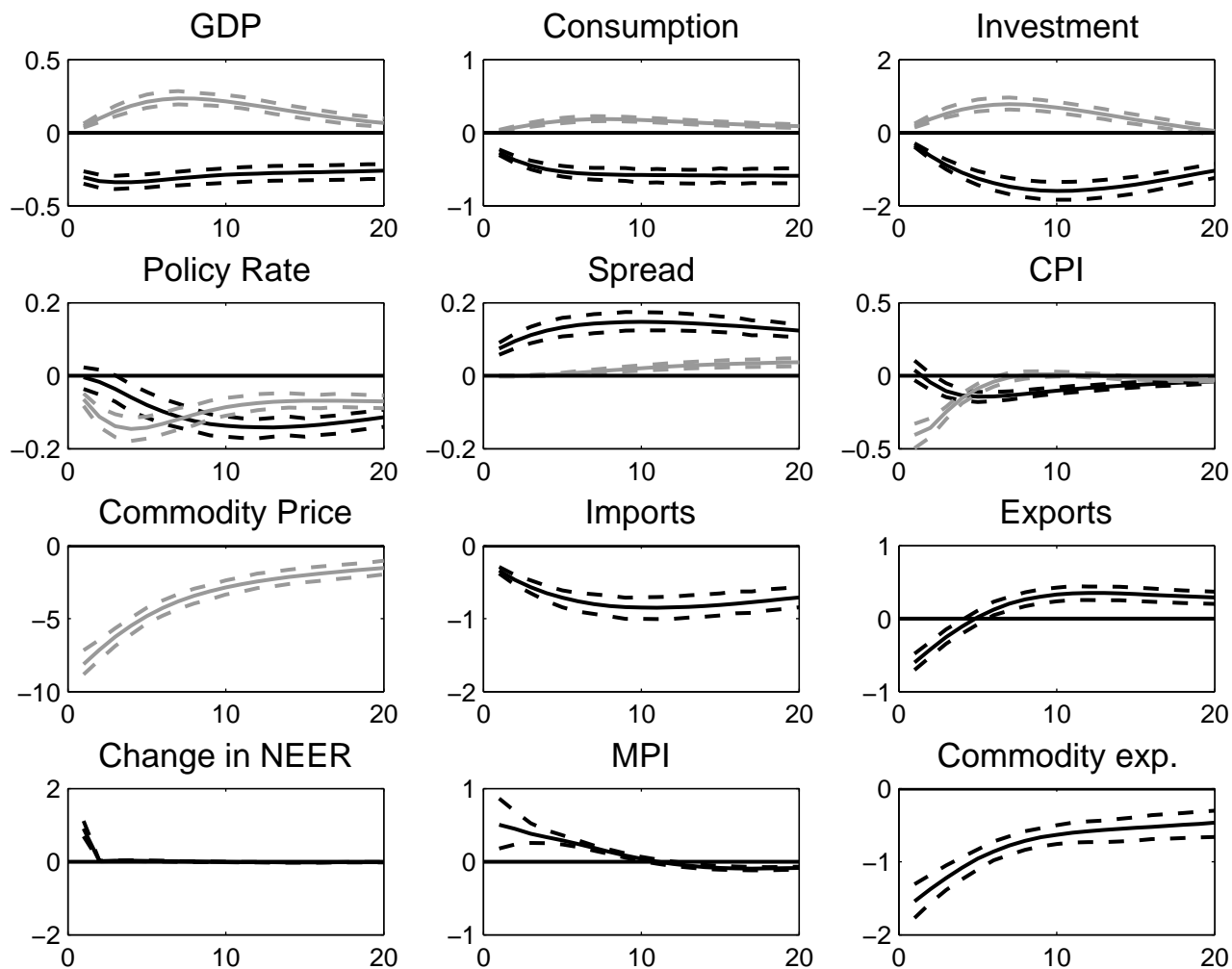


Figure 11: IRFs - Foreign Consumption Demand shock

Note: Variables expressed in percentage deviation from steady-state, inflation, spread and interest rates annualized. Horizon in quarters. Baseline model with SA variables in black and US variables in grey and 90% confidence bands.

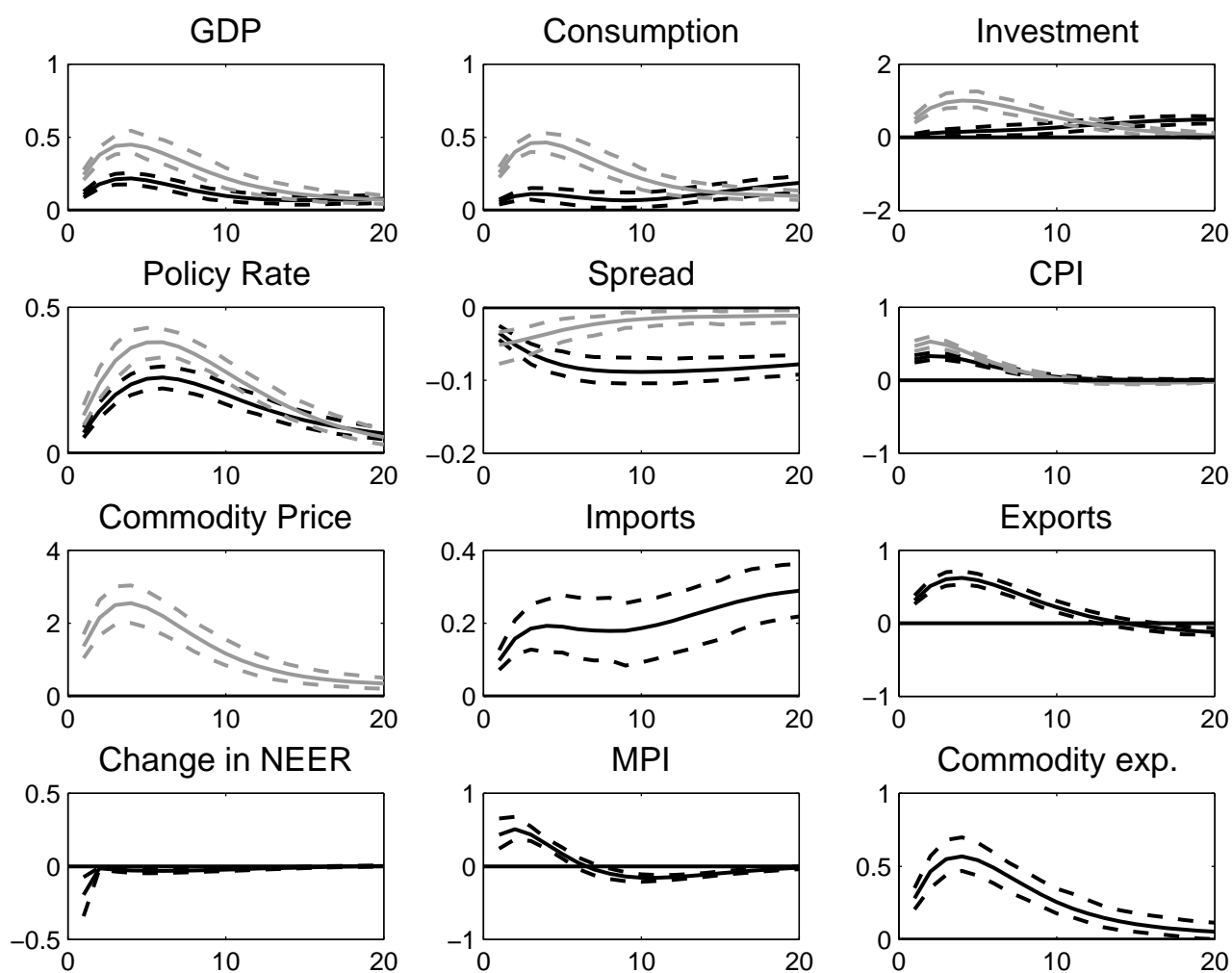


Figure 12: IRFs - Foreign Investment Demand shock

Note: Variables expressed in percentage deviation from steady-state, inflation, spread and interest rates annualized. Horizon in quarters. Baseline model with SA variables in black and US variables in grey and 90% confidence bands.

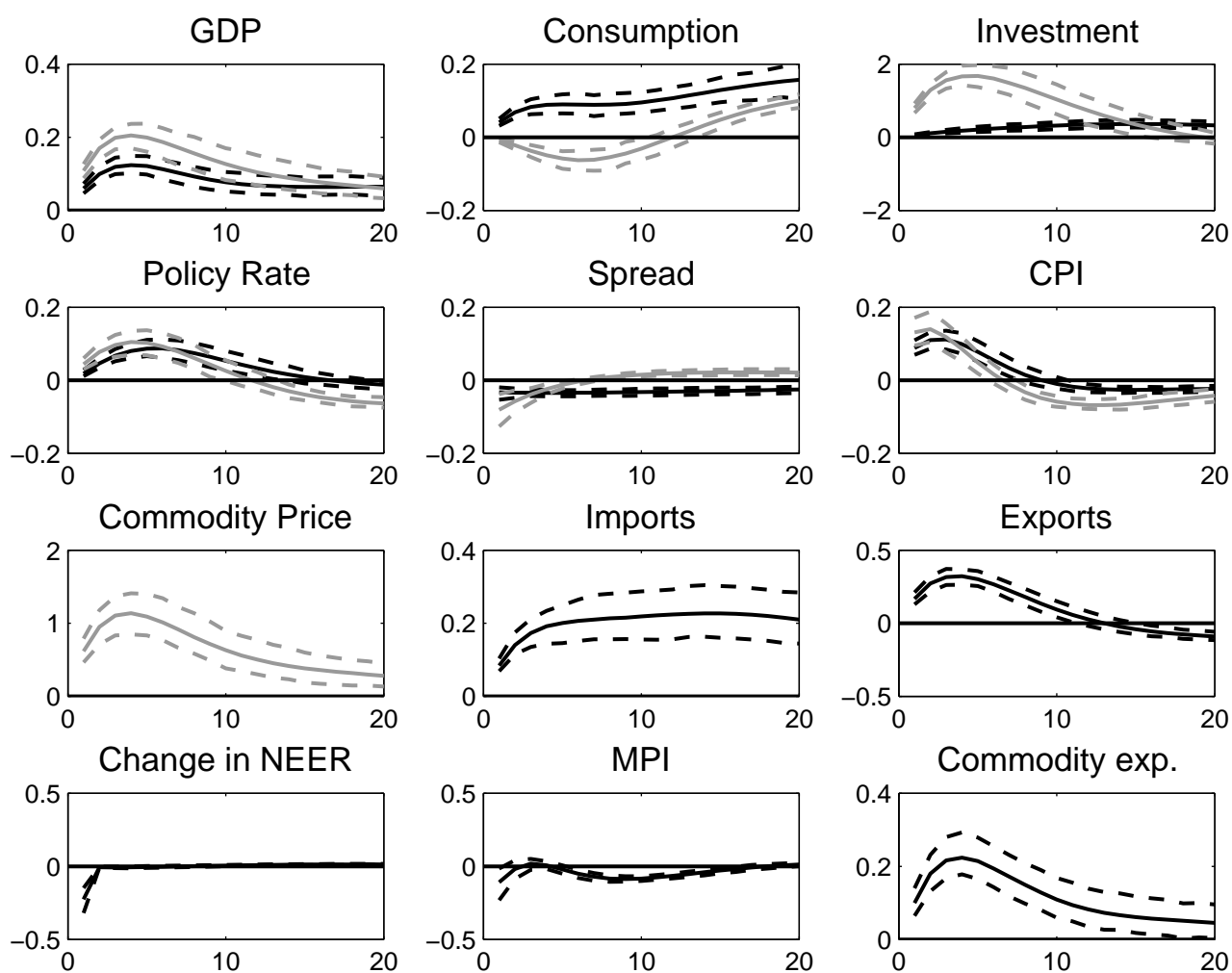


Figure 13: IRFs - Foreign Government Demand shock

Note: Variables expressed in percentage deviation from steady-state, inflation, spread and interest rates annualized. Horizon in quarters. Baseline model with SA variables in black and US variables in grey and 90% confidence bands.

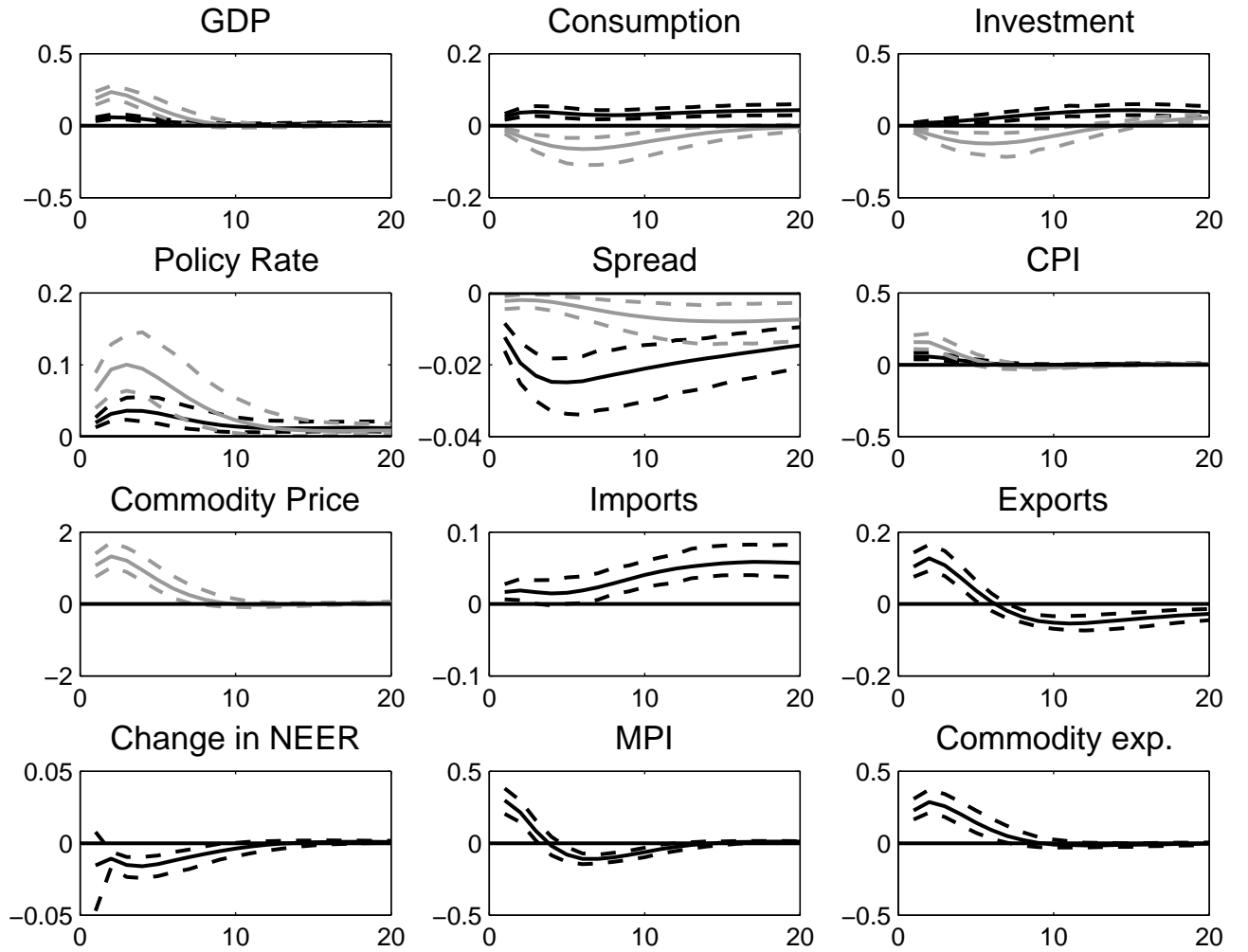


Figure 14: IRFs - Foreign Supply (cost-push) shock

Note: Variables expressed in percentage deviation from steady-state, inflation, spread and interest rates annualized. Horizon in quarters. Baseline model with SA variables in black and US variables in grey and 90% confidence bands.

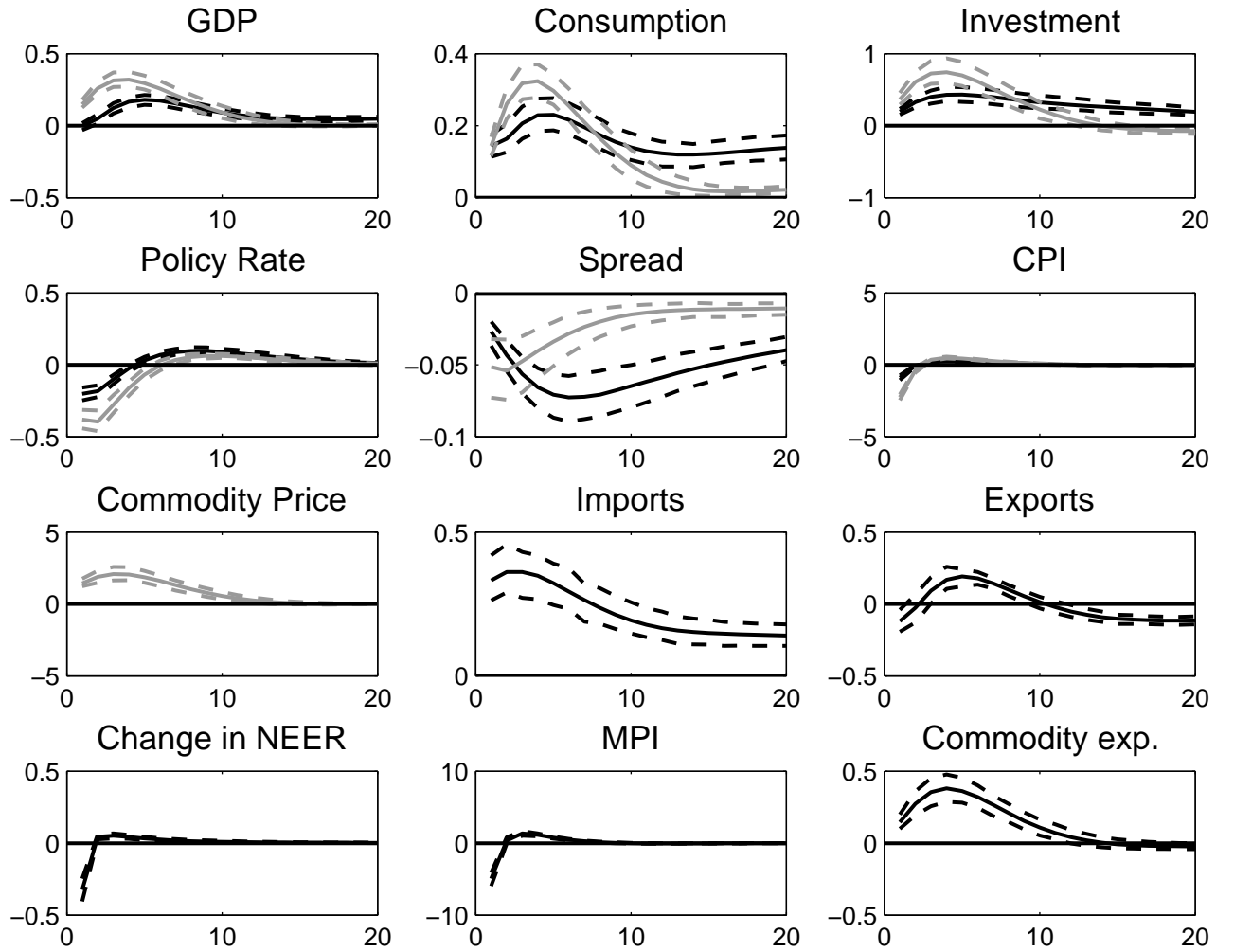


Figure 15: IRFs - Foreign Supply (productivity) shock

Note: Variables expressed in percentage deviation from steady-state, inflation, spread and interest rates annualized. Horizon in quarters. Baseline model with SA variables in black and US variables in grey and 90% confidence bands.

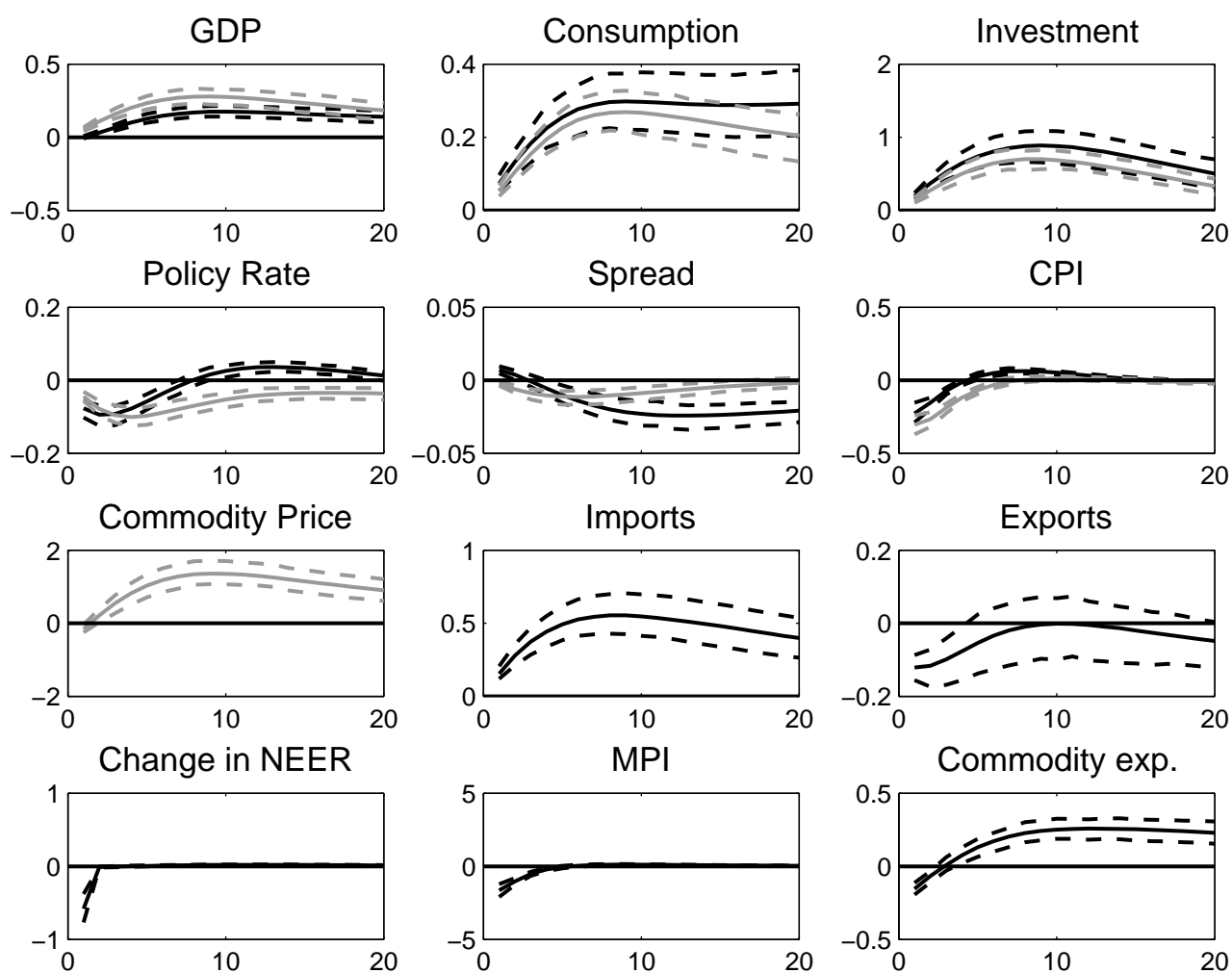


Figure 16: IRFs - Foreign Monetary Policy shock

Note: Variables expressed in percentage deviation from steady-state, inflation, spread and interest rates annualized. Horizon in quarters. Baseline model with SA variables in black and US variables in grey and 90% confidence bands.

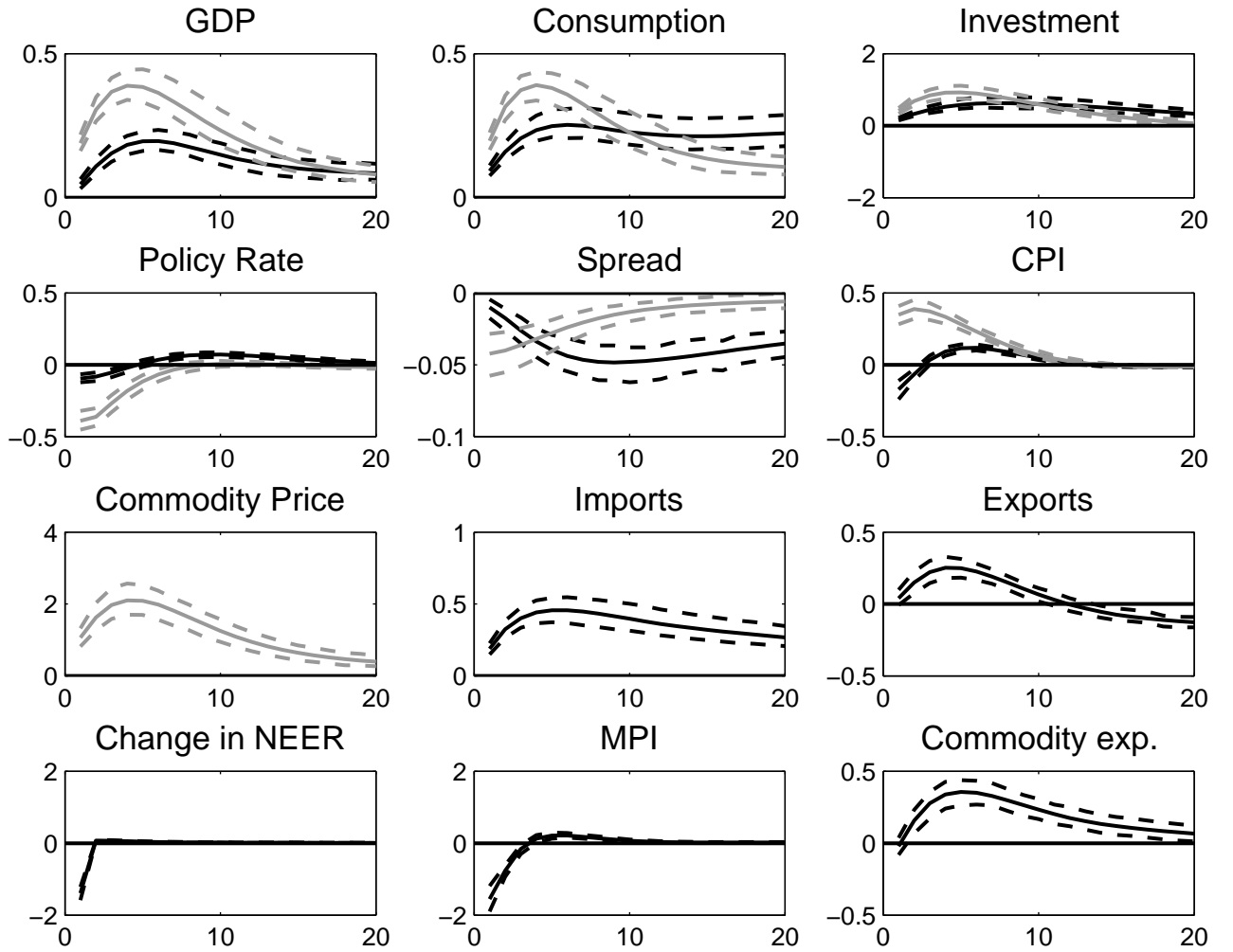


Figure 17: IRFs - Foreign Credit supply shock

Note: Variables expressed in percentage deviation from steady-state, inflation, spread and interest rates annualized. Horizon in quarters. Baseline model with SA variables in black and US variables in grey and 90% confidence bands.

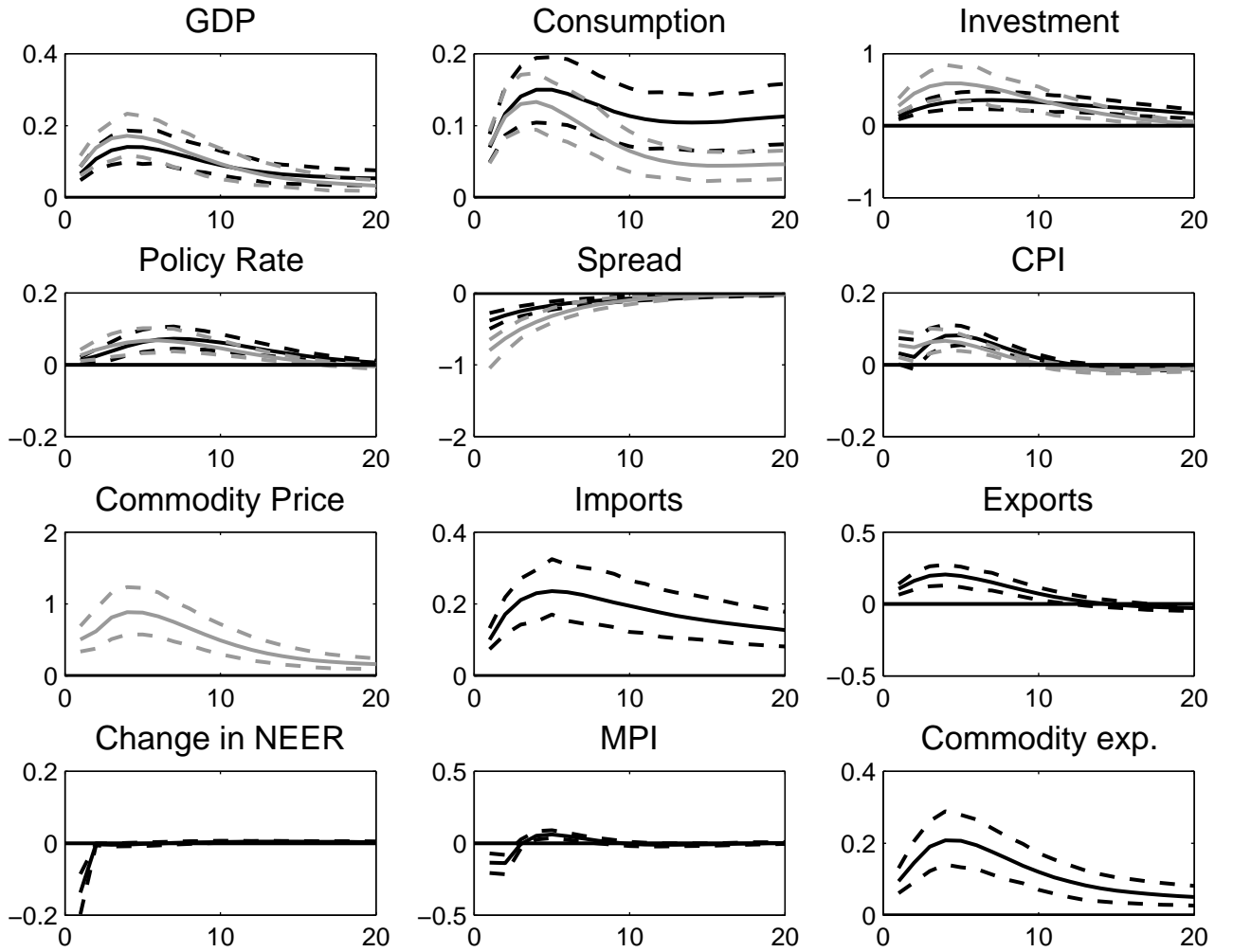


Figure 18: IRFs - Import Price shock

Note: Variables expressed in percentage deviation from steady-state, inflation, spread and interest rates annualized. Horizon in quarters. Baseline model with SA variables in black and US variables in grey and 90% confidence bands.

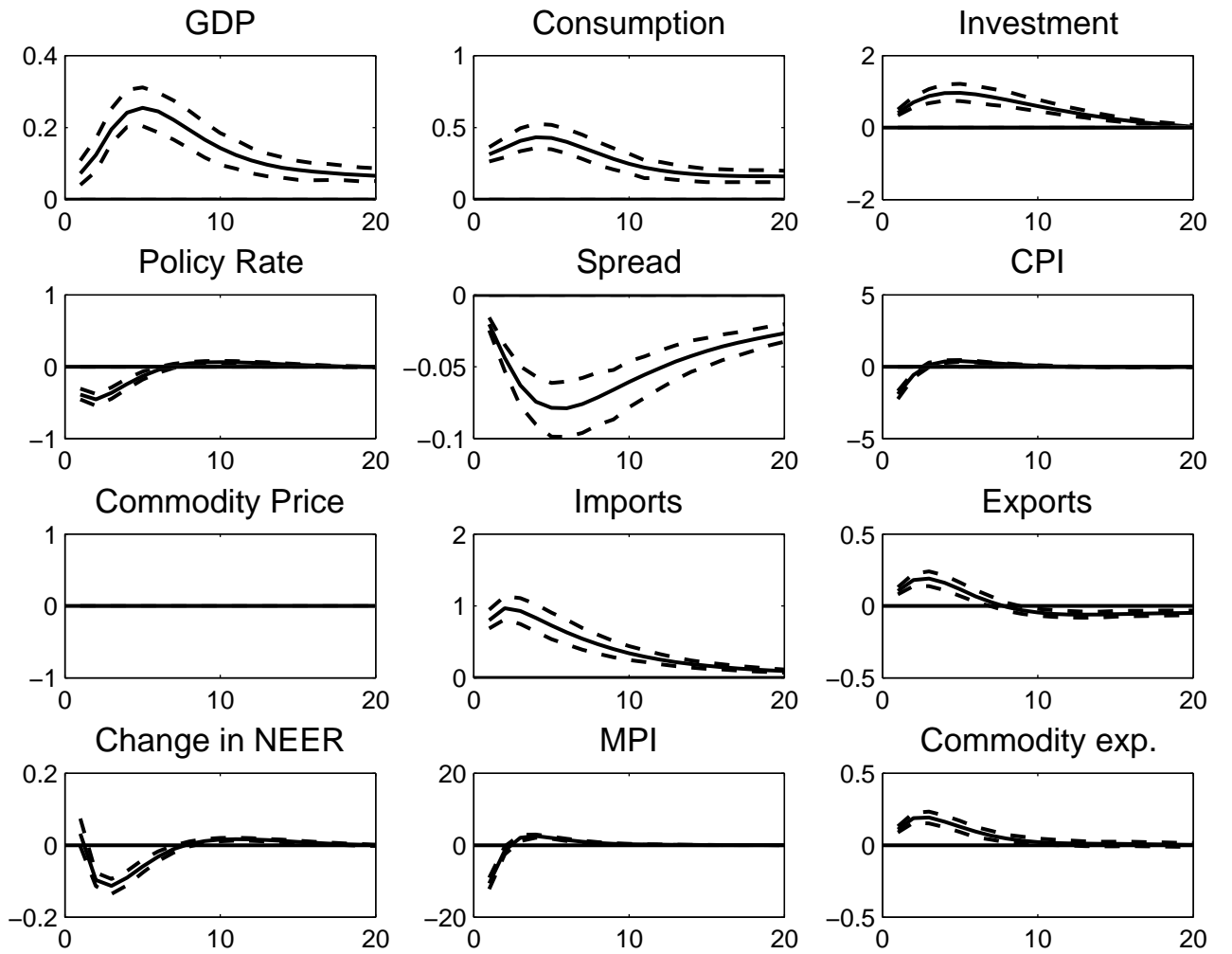


Figure 19: IRFs - Export Demand shock

Note: Variables expressed in percentage deviation from steady-state, inflation, spread and interest rates annualized. Horizon in quarters. Baseline model with SA variables in black and US variables in grey and 90% confidence bands.

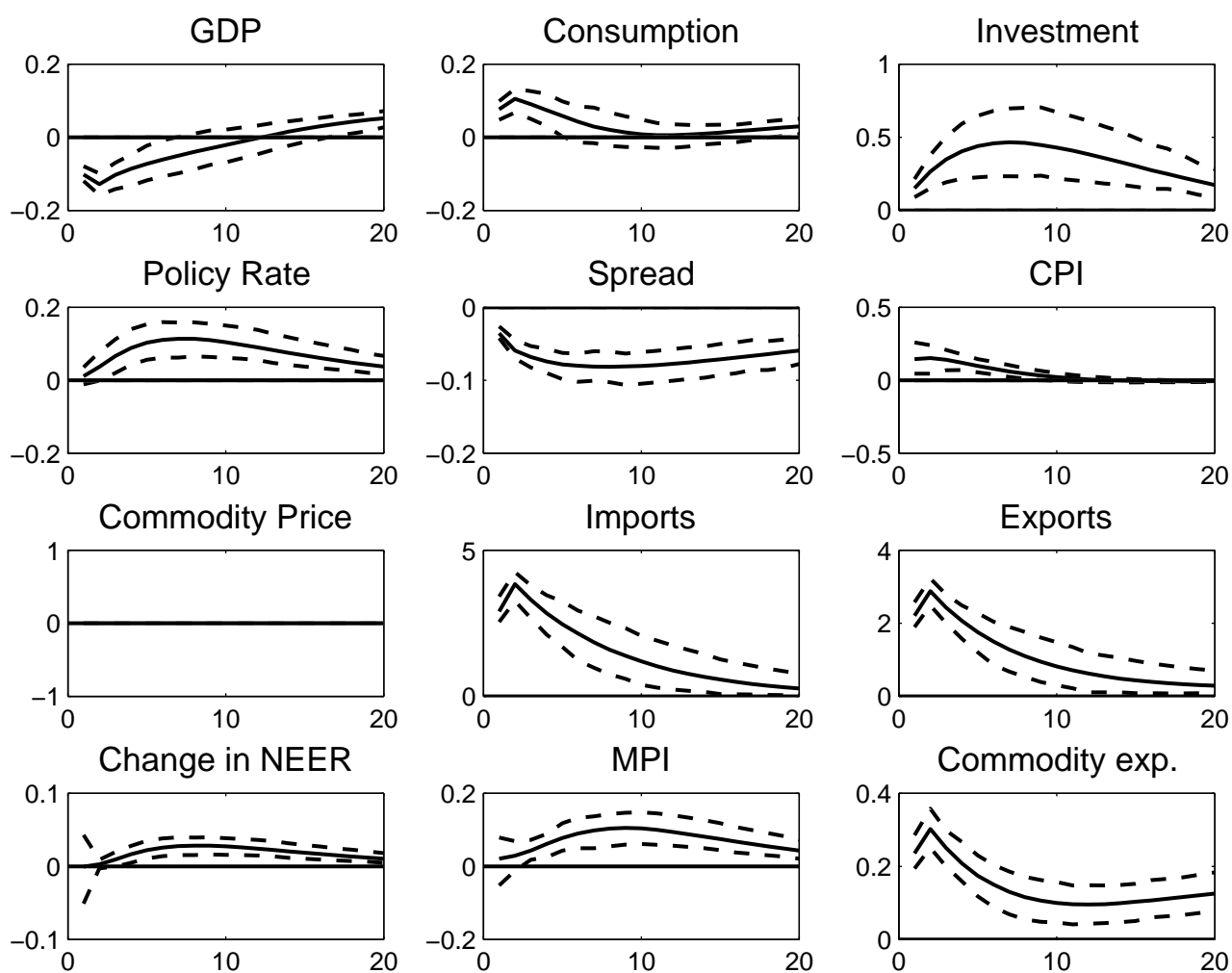


Figure 20: IRFs - Trade shock

Note: Variables expressed in percentage deviation from steady-state, inflation, spread and interest rates annualized. Horizon in quarters. Baseline model with SA variables in black and US variables in grey and 90% confidence bands.

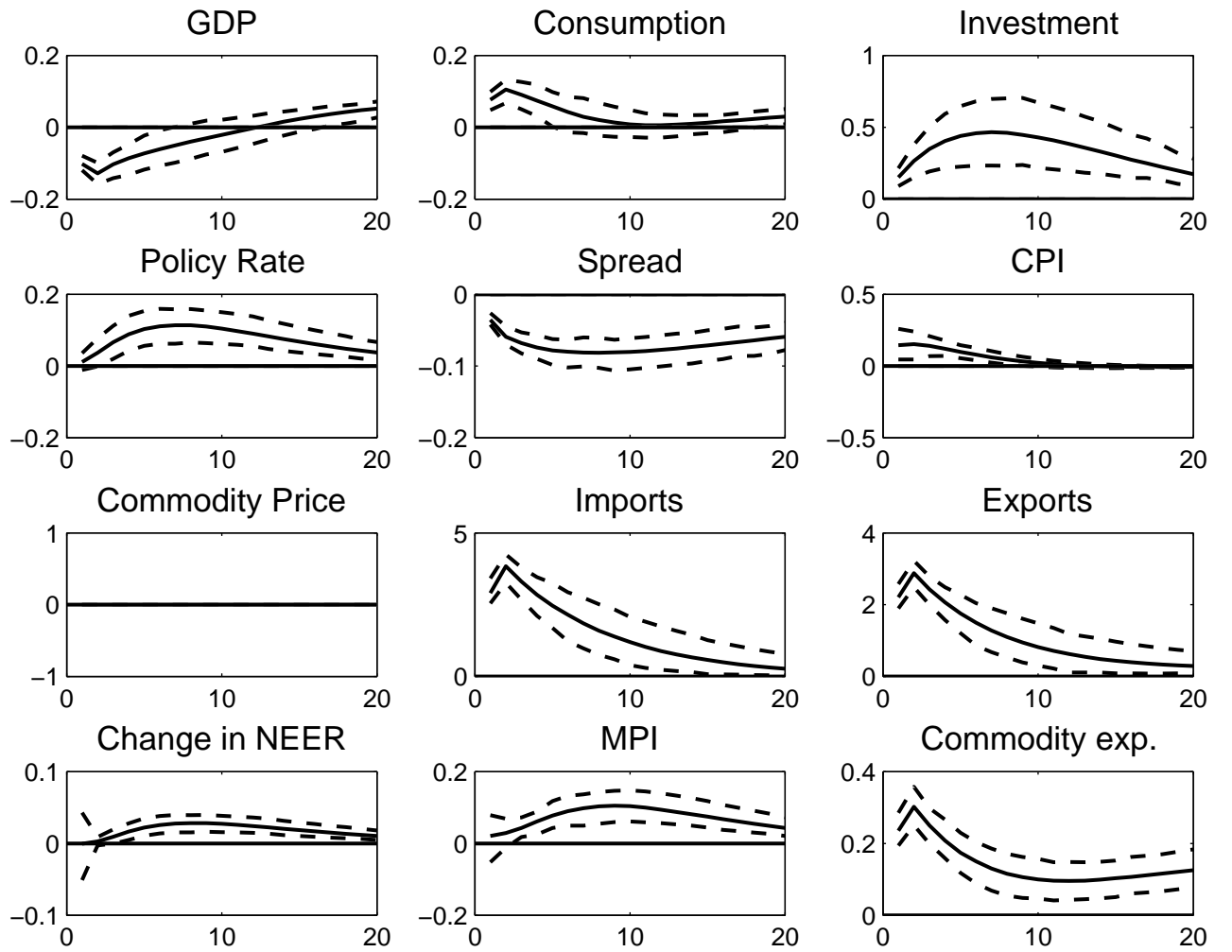


Figure 21: IRFs - UIP shock

Note: Variables expressed in percentage deviation from steady-state, inflation, spread and interest rates annualized. Horizon in quarters. Baseline model with SA variables in black and US variables in grey and 90% confidence bands.

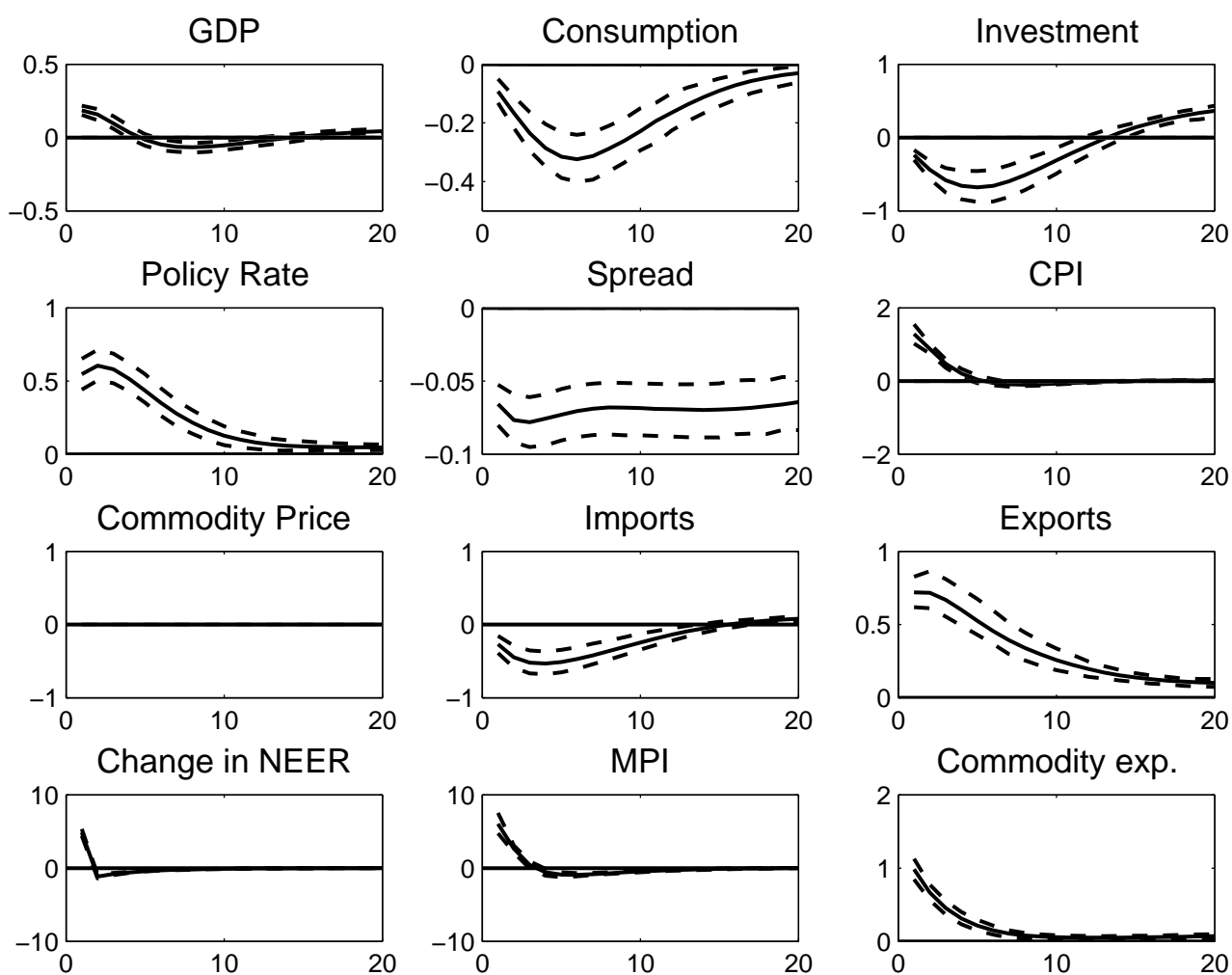


Figure 22: IRFs - Domestic Consumption Demand shock

Note: Variables expressed in percentage deviation from steady-state, inflation, spread and interest rates annualized. Horizon in quarters. Baseline model with SA variables in black and US variables in grey and 90% confidence bands.

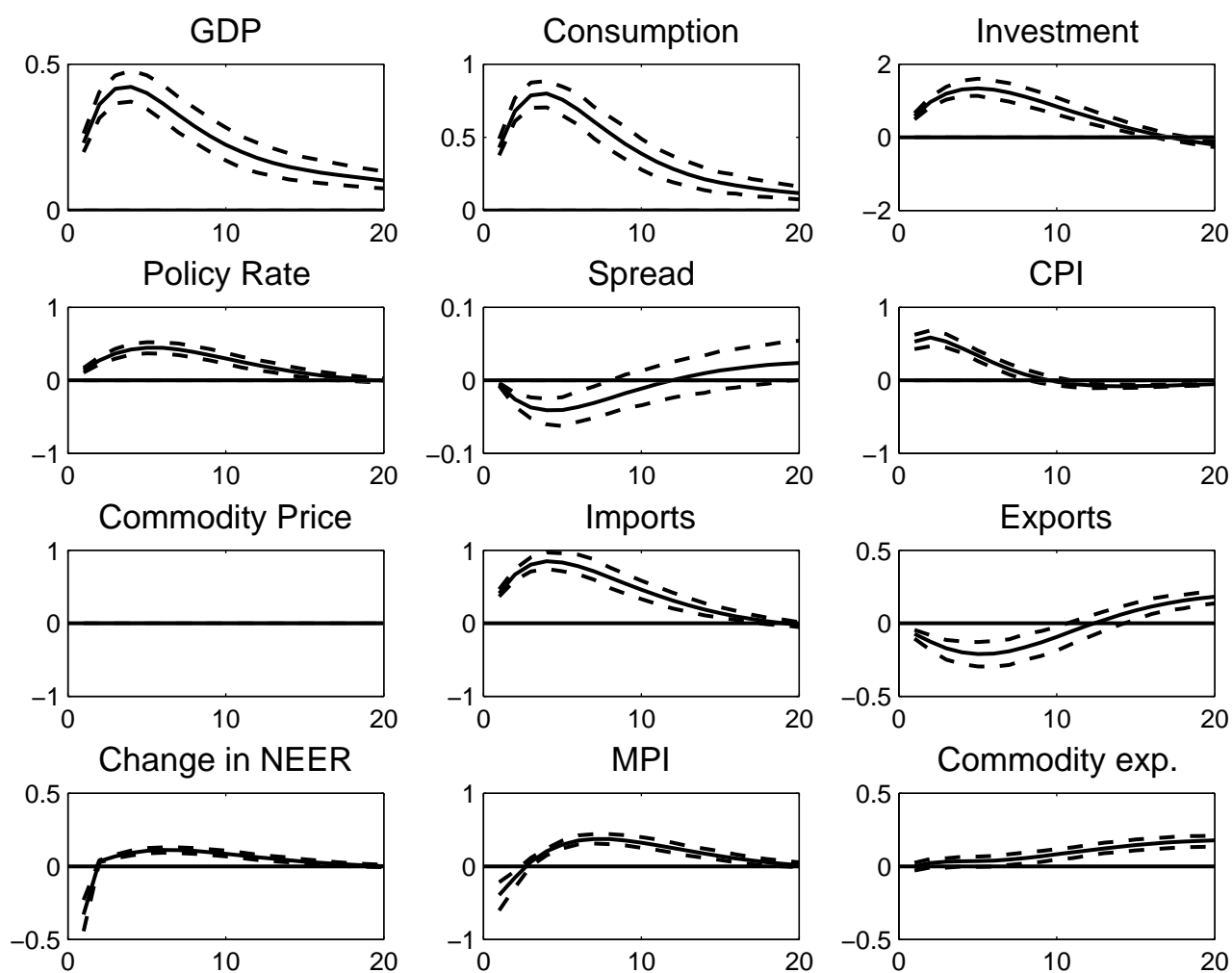


Figure 23: IRFs - Domestic Investment Demand shock

Note: Variables expressed in percentage deviation from steady-state, inflation, spread and interest rates annualized. Horizon in quarters. Baseline model with SA variables in black and US variables in grey and 90% confidence bands.

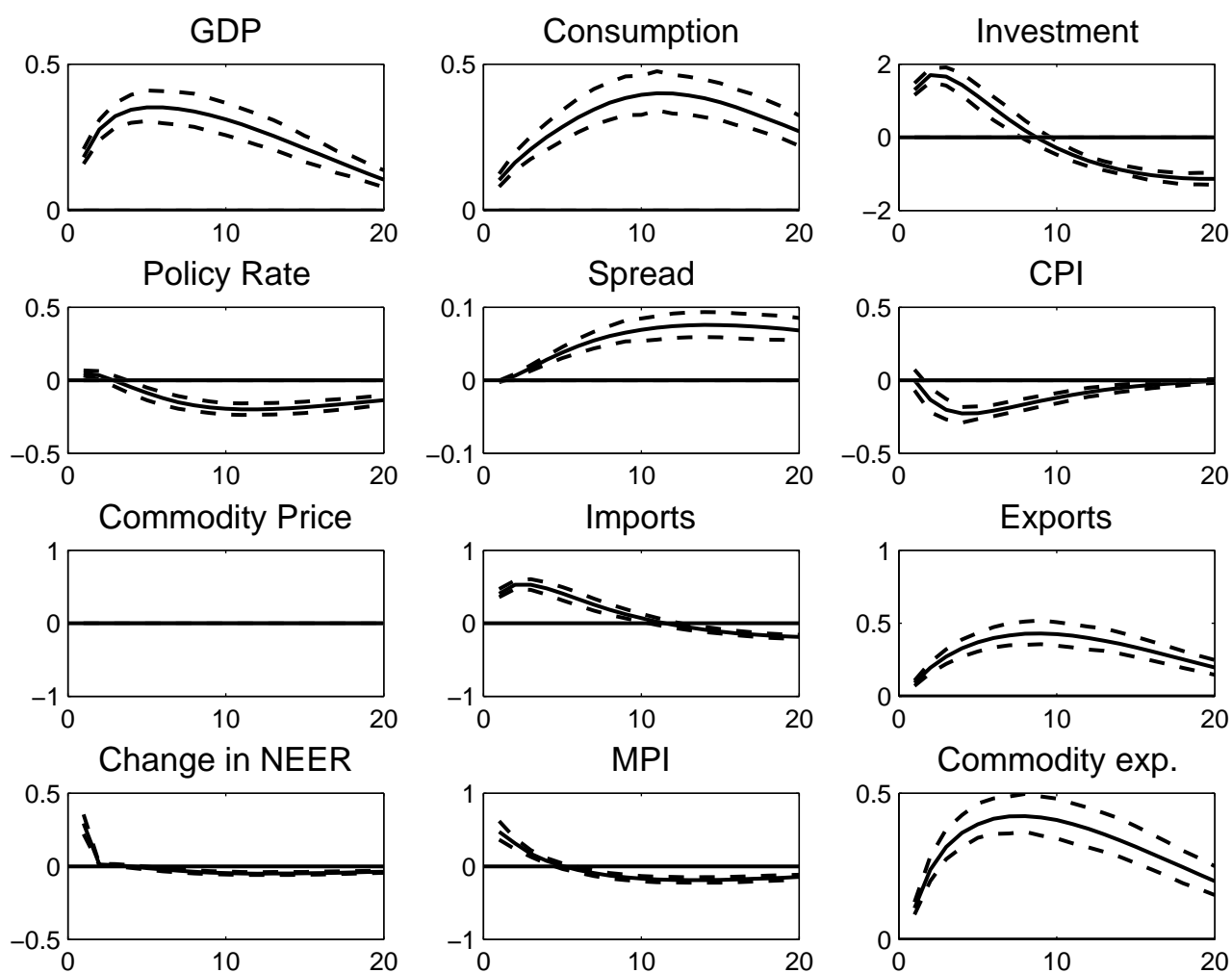


Figure 24: IRFs - Domestic Government Demand shock

Note: Variables expressed in percentage deviation from steady-state, inflation, spread and interest rates annualized. Horizon in quarters. Baseline model with SA variables in black and US variables in grey and 90% confidence bands.

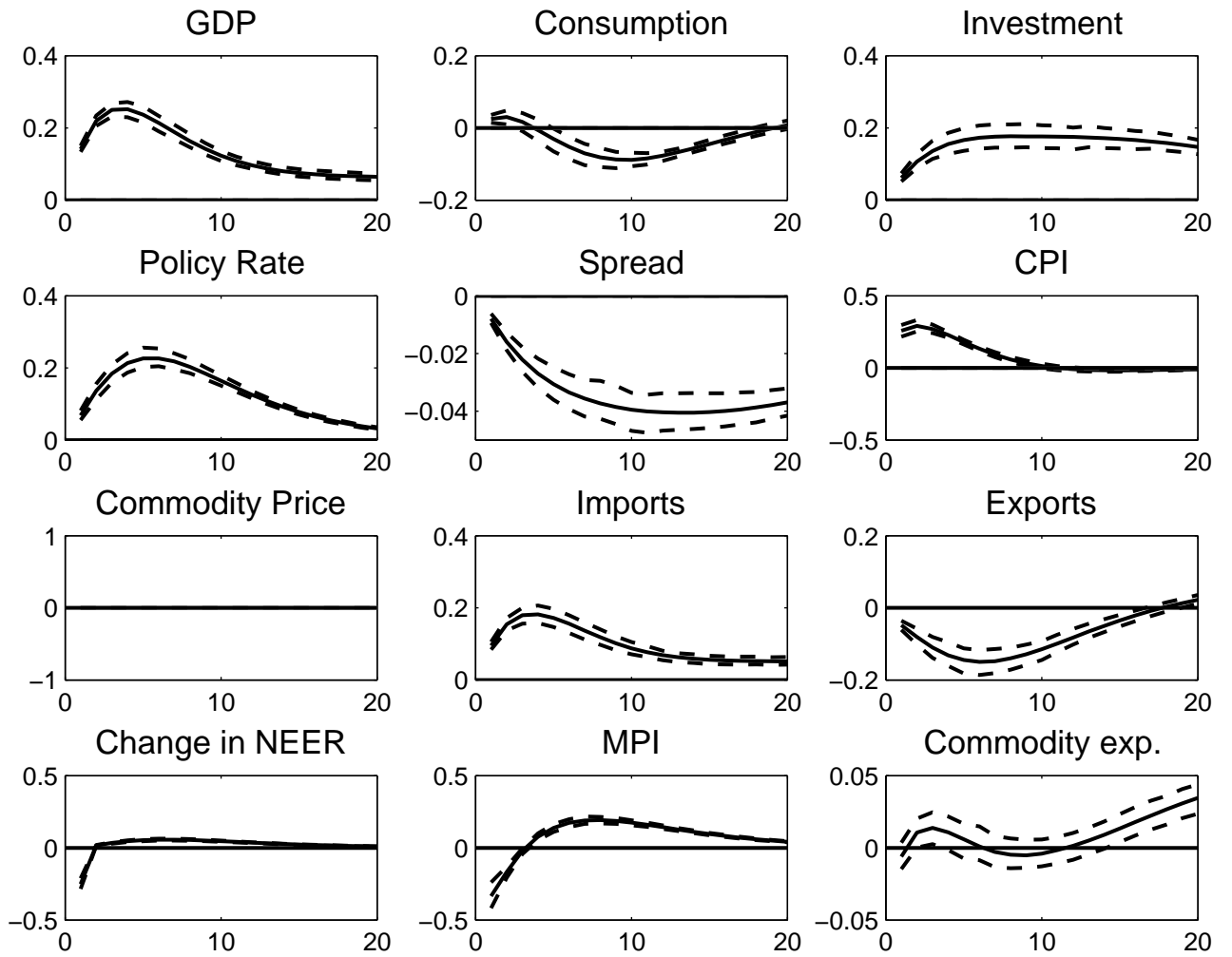


Figure 25: IRFs - Domestic Supply (productivity) shock

Note: Variables expressed in percentage deviation from steady-state, inflation, spread and interest rates annualized. Horizon in quarters. Baseline model with SA variables in black and US variables in grey and 90% confidence bands.

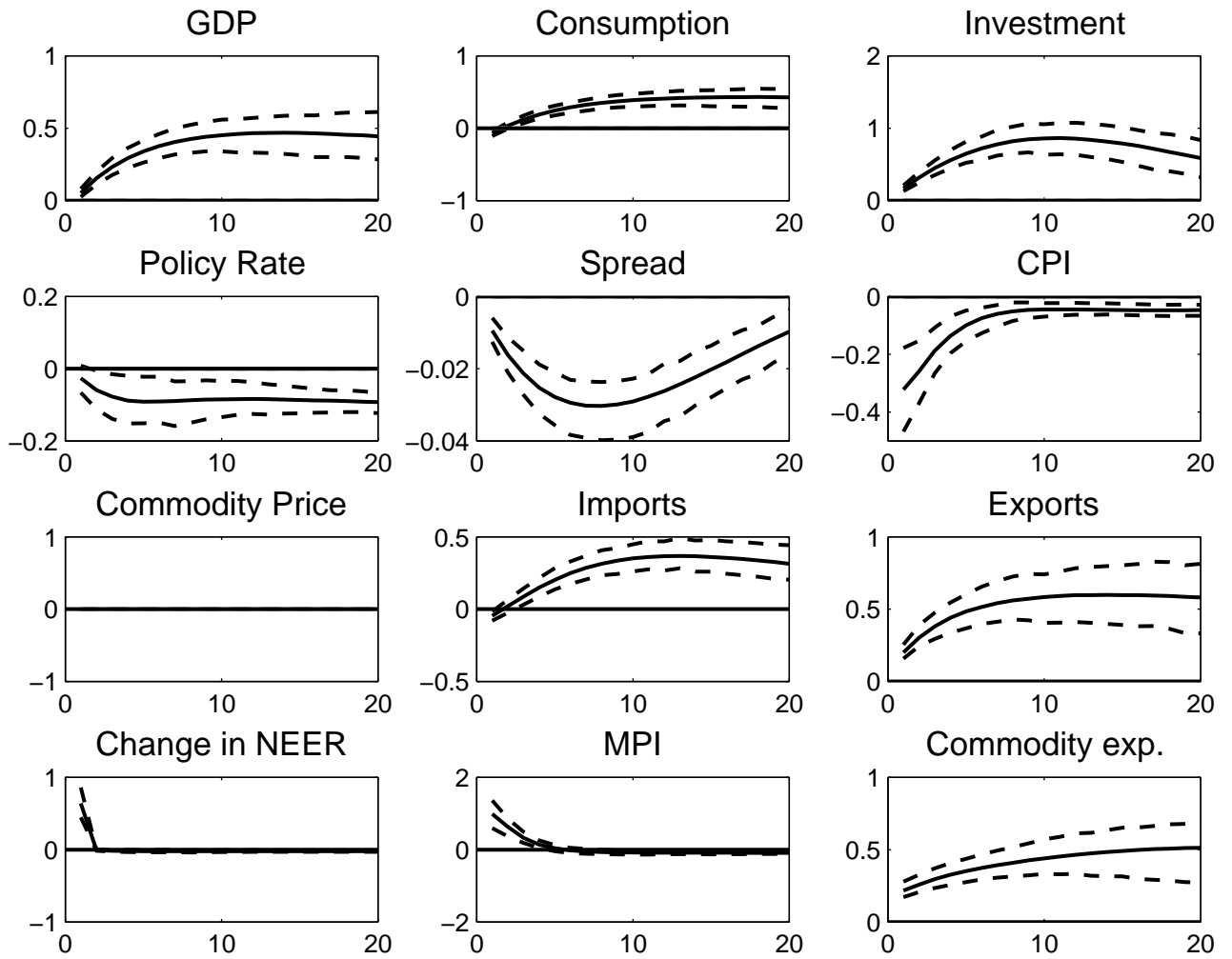


Figure 26: IRFs - Domestic Supply (cost-push) shock

Note: Variables expressed in percentage deviation from steady-state, inflation, spread and interest rates annualized. Horizon in quarters. Baseline model with SA variables in black and US variables in grey and 90% confidence bands.

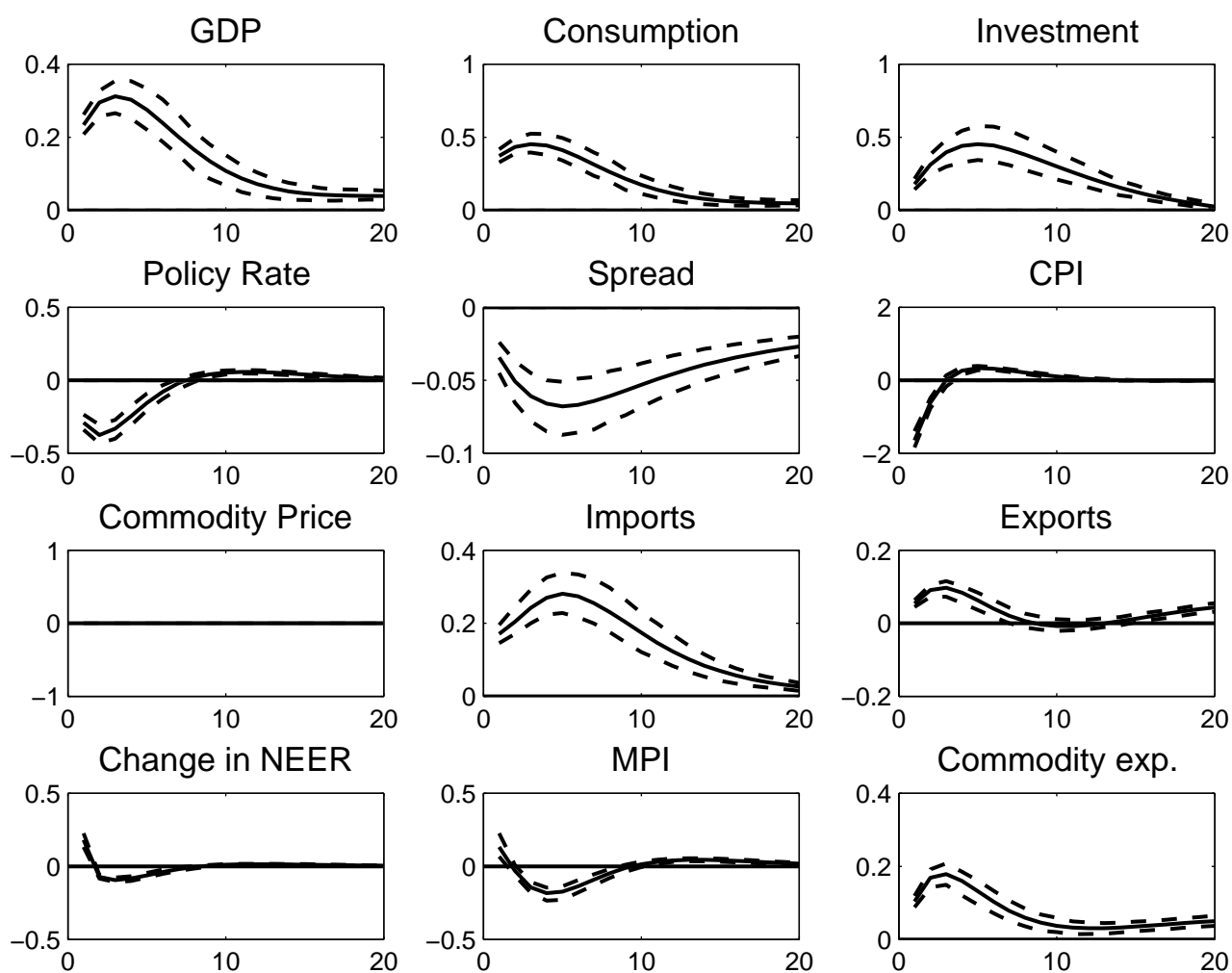


Figure 27: IRFs - Domestic Monetary Policy shock

Note: Variables expressed in percentage deviation from steady-state, inflation, spread and interest rates annualized. Horizon in quarters. Baseline model with SA variables in black and US variables in grey and 90% confidence bands.

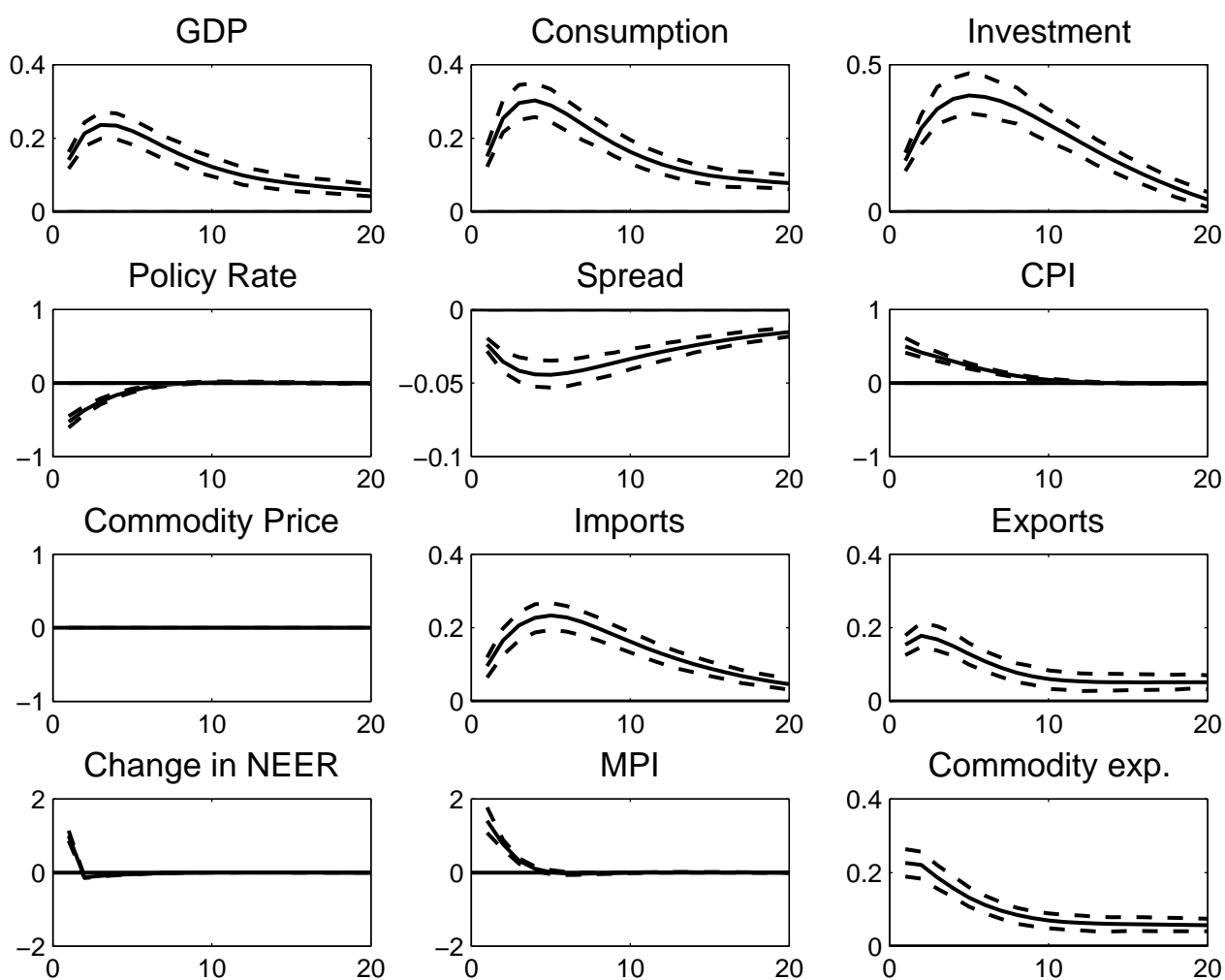


Figure 28: IRFs - Domestic Credit Supply shock

Note: Variables expressed in percentage deviation from steady-state, inflation, spread and interest rates annualized. Horizon in quarters. Baseline model with SA variables in black and US variables in grey and 90% confidence bands.

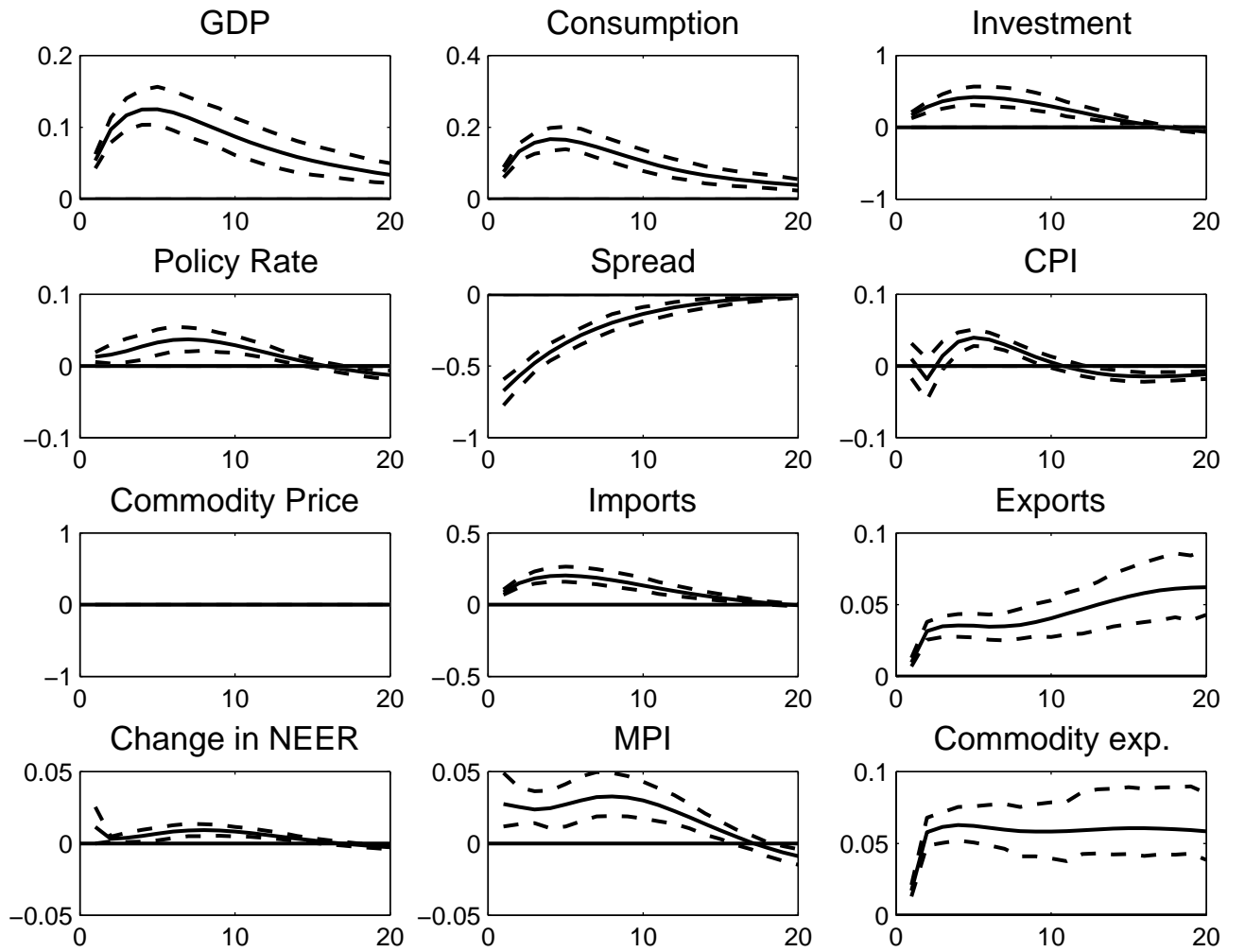
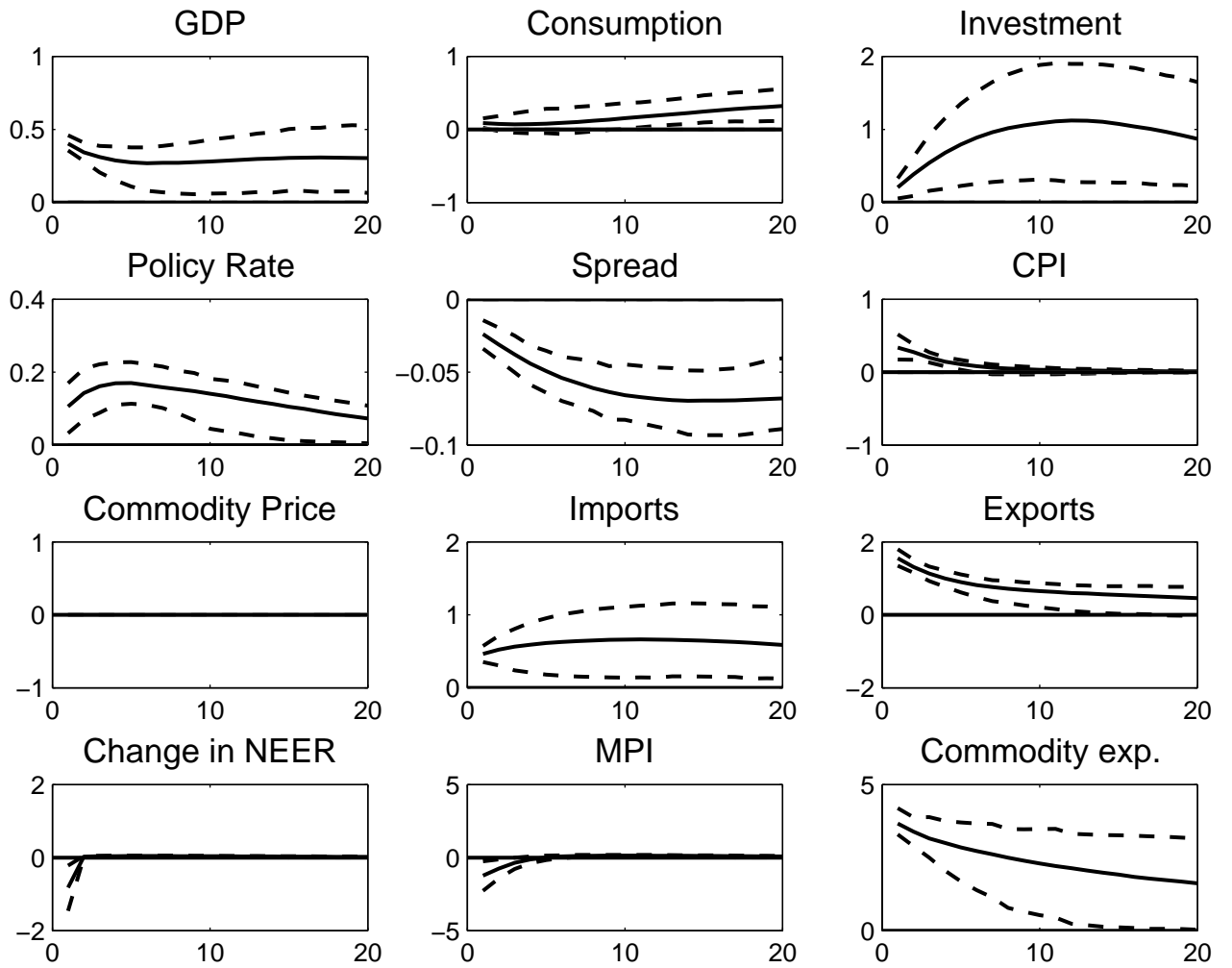


Figure 29: IRFs - Domestic Commodity Supply shock

Note: Variables expressed in percentage deviation from steady-state, inflation, spread and interest rates annualized. Horizon in quarters. Baseline model with SA variables in black and US variables in grey and 90% confidence bands.



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